Write your name here

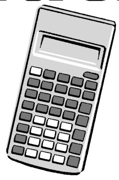
Surname	Other names

Pearson Edexcel
International GCSE

Centre Number | Candidate Number

Further Pure Mathematics

Level 2
Paper 2



Sample assessment material for first teaching September 2017
Time: 2 hours

Paper Reference
4PM1/02

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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PEARSON

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (diferentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The nth term of a geometric series is $3e^{(1-2n)}$

Find the sum to infinity of this series.

Give your answer in the form $\dfrac{ae}{e^b - 1}$ where a and b are integers to be found.

(5)

(Total for Question 1 is 5 marks)

2 Find the set of values of x for which

(a) $3 + x < 2x - 1$

(1)

(b) $x(x - 1) > 6$

(3)

(c) **both** $3 + x < 2x - 1$ **and** $x(x - 1) > 6$

(1)

(Total for Question 2 is 5 marks)

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3 O, A and B are fixed points such that

$$\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j} \qquad \overrightarrow{OB} = 8\mathbf{i} + p\mathbf{j} \qquad \text{and} \qquad \left|\overrightarrow{AB}\right| = 2\sqrt{13}$$

(a) Find the possible values of p.

(3)

Given that $p > 0$

(b) find a unit vector parallel to $\overrightarrow{AB}$

(2)

(Total for Question 3 is 5 marks)

4 $$f(x) = 2x^3 + px^2 + qx + 12 \quad p, q \in \mathbb{Z}$$

Given that $(x + 3)$ is a factor of $f(x)$ and that when $f'(x)$ is divided by $(x + 3)$ the remainder is 37

(a) show that $p = 1$ and find the value of q

(6)

(b) hence factorise $f(x)$ completely

(2)

(c) show that the equation $f(x) = 0$ has only one real root.

(2)

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Question 4 continued

(Total for Question 4 is 10 marks)

5 (a) Show that $\cos(A - B) - \cos(A + B) = 2\sin A \sin B$

(2)

(b) Hence express $2\sin 5x \sin 3x$ in the form $\cos mx - \cos nx$ where m and n are integers, giving the value of m and the value of n,

(1)

(c) (i) Find $\int 4\sin 5\theta \sin 3\theta \, d\theta$

(ii) Hence evaluate $\int_0^{\frac{\pi}{6}} 4\sin 5\theta \sin 3\theta \, d\theta$, giving your answer in the form $\frac{a\sqrt{b}}{c}$ where a, b and c are integers.

(4)

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Question 5 continued

(Total for Question 5 is 7 marks)

6 Solve the equation $\log_2 x + 6\log_x 2 = 7$

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Question 6 continued

(Total for Question 6 is 7 marks)

7 The curve C with equation

$$y = \frac{ax - 5}{x - b}$$

where a and b are integers, crosses the x-axis at the point (2.5, 0). The asymptote to C which is parallel to the y-axis has equation $x = 1$

(a) (i) Show that $a = 2$

(ii) Find the value of b.

(3)

(b) Find the coordinates of the point where C crosses the y-axis.

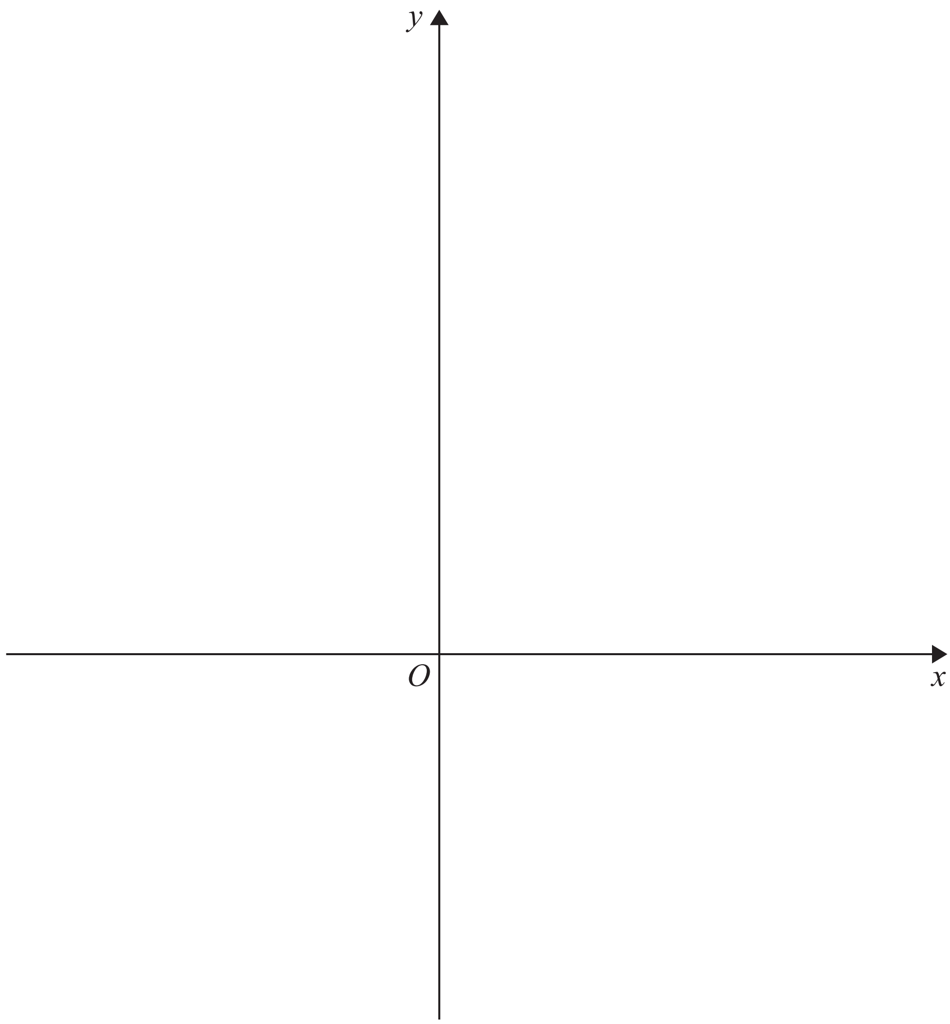
(1)

(c) Find the equation of the asymptote to C which is parallel to the x-axis.

(1)

(d) Using the axes below, sketch the curve C showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)



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Question 7 continued

(Total for Question 7 is 8 marks)

8 (a) Expand $\dfrac{3}{\sqrt{1-2x}}$ in ascending powers of x up to and including the term in x^3 and simplifying each term as far as possible.

(4)

(b) Write down the range of values of x for which this expansion is valid.

(1)

(c) Show that $\dfrac{3}{\sqrt{0.9}} = \sqrt{10}$

(1)

(d) Express $\dfrac{1}{\sqrt{10}-3}$ in the form $a\sqrt{10}+b$, where a and b are integers.

(2)

(e) Hence, using your expansion with a suitable value for x, obtain an approximation to 5 decimal places of $\dfrac{1}{\sqrt{10}-3}$

(3)

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Question 8 continued

(Total for Question 8 is 11 marks)

9 $$f(x) = 7 + 4x - 2x^2$$

Given that $f(x)$ can be written in the form $P(x + Q)^2 + R$ where P, Q and R are constants,

(a) find the value of P, the value of Q and the value of R.

(3)

(b) hence write down

(i) the maximum value of $f(x)$,

(ii) the value of x for which this maximum occurs.

(2)

The curve C has equation $y = 7 + 4x - 2x^2$

The line l with equation $y = 4 - x$ intersects C at two points.

(c) Find the x coordinates of these two points.

(3)

The finite region bounded by the curve C and the line l is rotated 360° about the x-axis.

(d) Use algebraic integration to find, to 3 significant figures, the volume of the solid generated.

(5)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 13 marks)

10

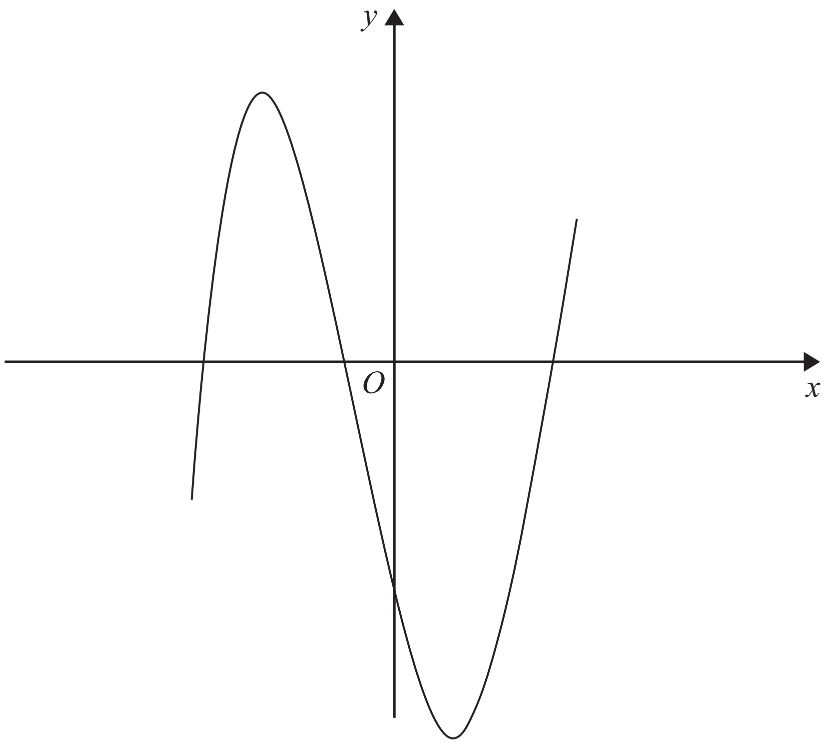

Figure 1

Figure 1 shows the curve M with equation $y = x^3 - 13x - 12$

The point P, with x coordinate -2, lies on M and line l_1 is the tangent to M at the point P.

(a) Find an equation for l_1

(5)

The point Q lies on M and the line l_2 is the tangent to M at the point Q.

Given that l_1 and l_2 are parallel,

(b) find an equation for l_2

(4)

The normal to M at P meets l_2 at the point R.

(c) Find the coordinates of R.

(4)

(d) Find the exact length of the line PR.

(2)

The tangent and normal at P and the tangent and normal at Q form a rectangle.

(e) Find the exact area of this rectangle.

(3)

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Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 18 marks)

11

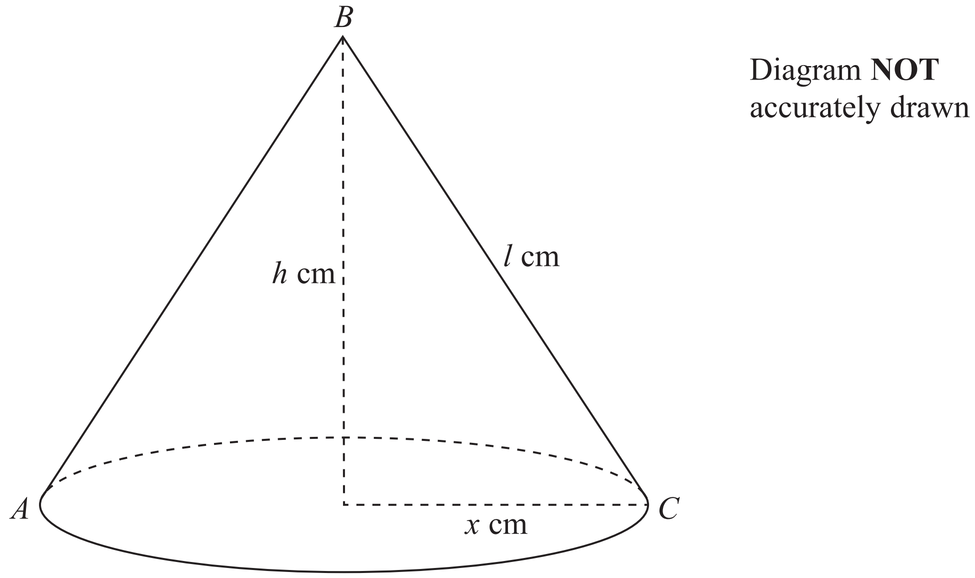

Figure 2

Figure 2 shows a right circular cone with a base radius of x cm. The slant height of the cone is l cm and the height of the cone is h cm. The vertex of the cone is B and the points A and C, on the base of the cone, are such that AC is a diameter of the base.

The cone is increasing in size in such a way that the size of the angle ABC is constant at 60° and the **total** surface area of the cone is increasing at a constant rate of 10 cm^2/s.

Find the exact rate of increase of the volume of the cone when $x = 6$

(11)

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Question 11 continued

Question 11 continued

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Question 11 continued

(Total for Question 11 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

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