Iteration and Numerical Methods Past Paper Answers GCSE Edexcel - Calculator

1.

x_1=-2.64 x_2= -2.57392 x_3=-2.603767255	M1 M1 A1	for substitution of −2.5 into the equation (to get $x_1 = -2.64$) for substitution of "$x_1 = -2.64$" and "$x_2 = -2.57392$" to give x_2 and x_3 for $x_1 = -2.64$ oe, $x_2 = -2.57(392)$ and $x_3 = -2.6(03767255)$ Condone $x_3 = -2.61$ if $x_2 = -2.57$ is used in the substitution
Statements	C1 C1	Connection between equation and iterative form in (a) e.g. rearrangement Statement e.g. iteration is an estimation of a solution

2.

(a)		Rearrangement	M1 for re arranging to $x^3 =$ C1 a clear step to show re arrangement
(b)	$x_1 = 3.29296875$ $x_2 = 3.276659786$ $x_3 = 3.279420684$	Values	M1 for substitution of 3.2 into the iterative formula A1 for $x_1 = 3.292(96875)$ A1 for $x_2 = 3.276(659786)$ and $x_3 = 3.279(420684)$
(c)		Statement	C1 Statement eg estimates of a solution to the original equation

3.

(a)	$F(x) = x^3 + 4x - 1$ $F(0) = -1$, $F(1) = 4$	Shown	M1 A1	Method to establish at least one root in [0,1]e.g $x^3 + 4x - 1$ (=0) and F(0)(= −1), F(1) (= 4) oe Since there is a sign change there must be at least one root in $0 < x < 1$ (as F is continuous)
(b)	$4x = 1 - x^3$ Or $\frac{x^3}{4} + x = \frac{1}{4}$	Shown	C1	C1 for at least one correct step and no incorrect ones
(c)	$x_1 = \frac{1}{4} - \frac{0}{4} = \frac{1}{4}$ $x_2 = \frac{1}{4} - \frac{\left(\frac{1}{4}\right)^3}{4} = \frac{1}{4} - \frac{1}{256}$	0.246(09375) Or $\frac{63}{256}$	B1 M1 A1	$x_1 = \frac{1}{4}$ M1 for $x_2 = \frac{1}{4} - \frac{\left(\frac{1}{4}\right)^3}{4}$ A1 for 0.246(09375) or $\frac{63}{256}$ oe

4.

(a)	Correct statement	C1	for substituting both 1 and 2 into $x^3 + x$ or into $x^3 + x - 7$	All arithmetic shown must be correct. Ignore any additional trials shown.
		C1	for values 2 and 10 plus explanation that these are above and below 7, or for values -5 and 3 plus explanation that there is a change of sign, thus implying a solution lies between 1 and 2	
(b)	Correct rearrangement	C1	for correct algebraic rearrangement	
(c)	1.74	M1	for substitution of 2 into the formula eg $\sqrt[3]{7-2}$ (= 1.70997…)	$x_1 = 1.70997\ldots$ $x_2 = 1.74241\ldots$ $x_3 = 1.73884\ldots$
		M1	for a substitution of x_1 to give x_2 (= 1.74241...)	Accept an accuracy of 2 dp or more rounded or truncated for values of x_1 and x_2
		A1	for answer in the range 1.738 to 1.74	Award the marks for 1.7 on the answer line provided correct iterations are shown in the working space.

5.

(a)		Shown	M1	for method to establish at least one root between $x = 0$ and $x = 1$, eg f(0) = – 5 and f(1) = 3
			C1	for correct values and a deduction about the roots eg as there is a sign change there must be at least one root between $x = 0$ and $x = 1$ (as f is continuous)
(b)		Shown	C1	for a correct first step in rearrangement, eg $x(x^2 + 7) - 5 = 0$ or $x^3 + 7x = 5$
			C1	for clear and correct steps showing complete rearrangement
(c)	$x_1 = 0.625$ $x_2 = 0.6765327696$ $x_3 = 0.6704483001$	0.6704(483001)	M1	for substitution of 1 into the formula (to get 0.625)
			M1	for substitution of "$x_1 = 0.625$" and "$x_2 = 0.6765327696$" to give x_2 and x_3
			A1	0.6704(483001)
(d)		Comment	M1	substitutes answer to (c) into expression (to get –0.00549...)
			C1	appropriate comment, eg accurate as answer is close to 0

6.

(a)	$2x^2 - 1 - \frac{4}{x} = 0$	Dividing equation by x	M1
	$x^2 = \frac{1}{2} + \frac{4}{2x}$	Obtaining $x^2 = \ldots$	M1
	$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$ *	cso	A1 **(3)**
(b)	$x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$		B1, B1, B1
If answers given to more than 2 dp, penalise first time then accept awrt above.			**(3)**

7.

	$f(x) = x^3 + 2x^2 - 3x - 11$		
(a)	$f(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$	Sets $f(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \frac{3x+11}{x+2}$		
	$\Rightarrow x = \sqrt{\left(\frac{3x+11}{x+2}\right)}$	then rearranges to give the quoted result on the question paper.	A1 AG (2
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$, $x_1 = 0$		
	$x_2 = \sqrt{\left(\frac{3(0)+11}{(0)+2}\right)}$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$ $x_4 = 2.058748112...$	Both x_2 = awrt 2.345 and x_3 = awrt 2.037 x_4 = awrt 2.059	A1 A1 (3
)	Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$		

8.

(*a*)	$x^2(3-x) - 1 = 0$ o.e. (e.g. $x^2(-x+3) = 1$) $x = \sqrt{\frac{1}{3-x}}$ (✱) Note(✱), answer is given: need to see appropriate working and A1 is cso [Reverse process: Squaring and non-fractional equation M1, form f(*x*) A1]	M1 A1 (cso) (2)
(*b*)	$x_2 = 0.6455$, $x_3 = 0.6517$, $x_4 = 0.6526$ 1st B1 is for one correct, 2nd B1 for other two correct If all three are to greater accuracy, award B0 B1	B1; B1 (2)

9.

(a)	$f(1.4) = -0.568 \ldots < 0$			
	$f(1.45) = 0.245 \ldots > 0$		M1	
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$		A1	(2)
(b)	$3x^3 = 2x + 6$			
	$x^3 = \frac{2x}{3} + 2$			
	$x^2 = \frac{2}{3} + \frac{2}{x}$		M1 A1	
	$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ *	cso	A1	(3)
(c)	$x_1 = 1.4371$		B1	
	$x_2 = 1.4347$		B1	
	$x_3 = 1.4355$		B1	(3)

10.

(a) $x^3 + 3x^2 + 4x - 12 = 0 \Rightarrow x^3 + 3x^2 = 12 - 4x$			
$\Rightarrow x^2(x+3) = 12 - 4x$		M1	
$\Rightarrow x^2 = \frac{12-4x}{(x+3)} \Rightarrow x = \sqrt{\frac{4(3-x)}{(x+3)}}$		dM1A1*	
			(3)
(b) $x_1 = 1.41$, *awrt* $x_2 = 1.20$ $x_3 = 1.31$		M1A1,A1	
			(3)