AS
Further Mathematics

Sample Assessment Materials

Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics (8FM0)

First teaching from September 2017

First certification from 2018

Issue 1

Edexcel, BTEC and LCCI qualifications

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Introduction

The Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics is designed for use in schools and colleges. It is part of a suite of AS/A Level qualifications offered by Pearson.

These sample assessment materials have been developed to support this qualification and will be used as the benchmark to develop the assessment students will take.

The booklet '*Mathematical Formulae and Statistical Tables'* will be provided for use with these assessments and can be downloaded from our website, qualifications.pearson.com.

General marking guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked unless the candidate has replaced it with an alternative response.

Specific guidance for mathematics

1. These mark schemes use the following types of marks:
    - M marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
    - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
    - B marks are unconditional accuracy marks (independent of M marks)
    - Marks should not be subdivided.
2. Abbreviations

    These are some of the traditional marking abbreviations that may appear in the mark schemes.

bod	benefit of doubt
ft	follow through
√	this symbol is used for correct ft
cao	correct answer only
cso	correct solution only. There must be no errors in this part of the question to obtain this mark
isw	ignore subsequent working
awrt	answers which round to
SC:	special case
o.e.	or equivalent (and appropriate)
d… or dep	dependent
indep	independent
dp	decimal places
sf	significant figures
*	The answer is printed on the paper or ag- answer given

- [or d… The second mark is dependent on gaining the first mark

3. All M marks are follow through.

   All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Paper 1: Core Pure Mathematics

Sample Assessment Material for first teaching September 2017

Time: 1 hour 40 minutes

Paper Reference
8FM0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. $$\mathrm{f}(z) = z^3 + pz^2 + qz - 15$$

where p and q are real constants.

Given that the equation $\mathrm{f}(z) = 0$ has roots

$$\alpha,\ \frac{5}{\alpha} \text{ and } \left(\alpha + \frac{5}{\alpha} - 1\right)$$

(a) solve completely the equation $\mathrm{f}(z) = 0$

(5)

(b) Hence find the value of p.

(2)

Question 1 continued

(Total for Question 1 is 7 marks)

2. The plane Π passes through the point A and is perpendicular to the vector $\mathbf{n}$

Given that

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix} \quad \text{and} \quad \mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

where O is the origin,

(a) find a Cartesian equation of Π .

(2)

With respect to the fixed origin O, the line l is given by the equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 3 \end{pmatrix}$$

The line l intersects the plane Π at the point X.

(b) Show that the acute angle between the plane Π and the line l is 21.2° correct to one decimal place.

(4)

(c) Find the coordinates of the point X.

(4)

Question 2 continued

(Total for Question 2 is 10 marks)

3. Tyler invested a total of £5000 across three different accounts; a savings account, a property bond account and a share dealing account.

Tyler invested £400 more in the property bond account than in the savings account.

After one year

- the savings account had increased in value by 1.5%
- the property bond account had increased in value by 3.5%
- the share dealing account had **decreased** in value by 2.5%

- the total value across Tyler's three accounts had increased by £79

Form and solve a matrix equation to find out how much money was invested by Tyler in each account.

(7)

Question 3 continued

(Total for Question 3 is 7 marks)

4. The cubic equation

$$x^3 + 3x^2 - 8x + 6 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

Question 4 continued

(Total for Question 4 is 5 marks)

5. $$\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

(a) Show that **M** is non-singular.

(2)

The hexagon R is transformed to the hexagon S by the transformation represented by the matrix **M**.

Given that the area of hexagon R is 5 square units,

(b) find the area of hexagon S.

(1)

The matrix **M** represents an enlargement, with centre (0, 0) and scale factor k, where $k > 0$, followed by a rotation anti-clockwise through an angle θ about (0, 0).

(c) Find the value of k.

(2)

(d) Find the value of θ.

(2)

Question 5 continued

(Total for Question 5 is 7 marks)

6. (a) Prove by induction that for all positive integers n,

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

(6)

(b) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r(r+6)(r-6) = \frac{1}{4}n(n+1)(n-8)(n+9)$$

(4)

(c) Hence find the value of n that satisfies

$$\sum_{r=1}^{n} r(r+6)(r-6) = 17\sum_{r=1}^{n} r^2$$

(5)

Question 6 continued

(Total for Question 6 is 15 marks)

7.

Diagrams not drawn to scale

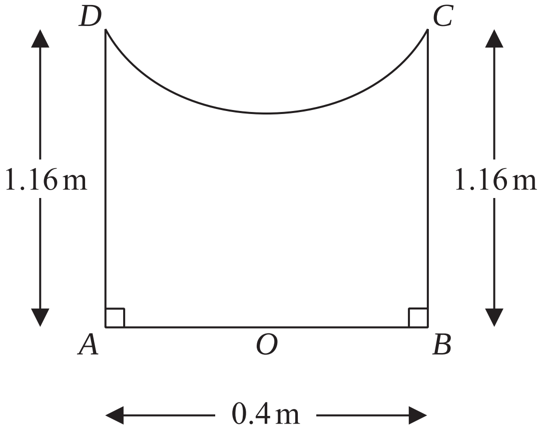


Figure 1

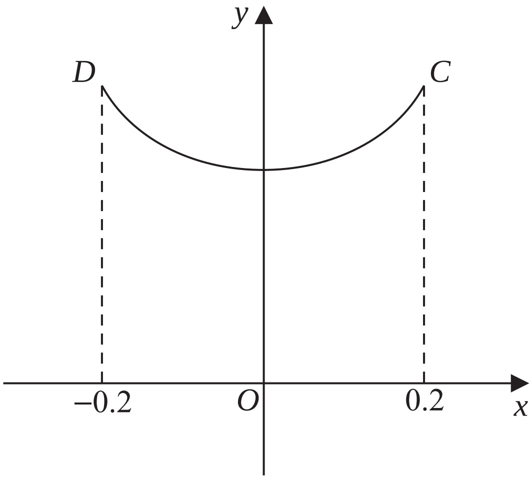


Figure 2

Figure 1 shows the central cross-section *AOBCD* of a circular bird bath, which is made of concrete. Measurements of the height and diameter of the bird bath, and the depth of the bowl of the bird bath have been taken in order to estimate the amount of concrete that was required to make this bird bath.

Using these measurements, the cross-sectional curve *CD*, shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2 \qquad -0.2 \leqslant x \leqslant 0.2$$

where k is a constant and where O is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter, *AB*, of the base of the bird bath measured 0.40 m, as shown in Figure 1.

(a) Suggest the maximum depth of the bird bath.

(1)

(b) Find the value of k.

(2)

(c) Hence find the volume of concrete that was required to make the bird bath according to this model. Give your answer, in m^3, correct to 3 significant figures.

(7)

(d) State a limitation of the model.

(1)

It was later discovered that the volume of concrete used to make the bird bath was $0.127\,m^3$ correct to 3 significant figures.

(e) Using this information and the answer to part (c), evaluate the model, explaining your reasoning.

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 12 marks)

8. (a) Shade on an Argand diagram the set of points

$$\left\{z \in \mathbb{C} : \left|z - 4\text{i}\right| \leqslant 3\right\} \cap \left\{z \in \mathbb{C} : -\frac{\pi}{2} < \arg(z + 3 - 4\text{i}) \leqslant \frac{\pi}{4}\right\}$$

(6)

The complex number w satisfies

$$|w - 4\text{i}| = 3$$

(b) Find the maximum value of $\arg w$ in the interval $(-\pi, \pi]$.
Give your answer in radians correct to 2 decimal places.

(2)

Question 8 continued

(Total for Question 8 is 8 marks)

9. An octopus is able to catch any fish that swim within a distance of 2 m from the octopus's position.

A fish F swims from a point A to a point B.

The octopus is modelled as a fixed particle at the origin O.

Fish F is modelled as a particle moving in a straight line from A to B.

Relative to O, the coordinates of A are $(-3, 1, -7)$ and the coordinates of B are $(9, 4, 11)$, where the unit of distance is metres.

(a) Use the model to determine whether or not the octopus is able to catch fish F.

(7)

(b) Criticise the model in relation to fish F.

(1)

(c) Criticise the model in relation to the octopus.

(1)

Question 9 continued

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 80 MARKS

Paper 1: Core Pure Mathematics Mark Scheme

Question	Scheme	Marks	AOs
1(a)	$\alpha\left(\frac{5}{\alpha}\right)\left(\alpha+\frac{5}{\alpha}-1\right)=15$	M1	1.1b
		A1	1.1b
	$\Rightarrow 5\alpha+\frac{25}{\alpha}-5=15 \Rightarrow \alpha^2-4\alpha+5=0$ $\Rightarrow \alpha=\frac{--4\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$ or $(\alpha-2)^2-4+5=0 \Rightarrow \alpha=...$	M1	3.1a
	$\Rightarrow \alpha=2\pm \mathrm{i}$	A1	1.1b
	Hence the roots of $\mathrm{f}(z)=0$ are $2+\mathrm{i}, 2-\mathrm{i}$ and 3	A1	2.2a
		(5)	
(b)	$p=-\left(\text{"}(2+\mathrm{i})\text{"}+\text{"}(2-\mathrm{i})\text{"}+\text{"}3\text{"}\right) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ cso	A1	1.1b
		(2)	
	1(b) alternative		
	$\mathrm{f}(z)=(z-3)(z^2-4z+5) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ cso	A1	1.1b
		(2)	
		(7 marks)	

Notes:

(a)

M1: Multiplies the three given roots together and sets the result equal to 15 or -15

A1: Obtains a correct equation in α

M1: Forms a quadratic equation in α and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha=....$

A1: $\alpha=2\pm \mathrm{i}$

A1: Deduces the roots are $2+\mathrm{i}, 2-\mathrm{i}$ and 3

(b)

M1: Applies the process of finding $-\sum\left(\text{of their three roots found in part }(a)\right)$ to give $p=...$

A1: $p=-7$ by correct solution only

(b) **Alternative**

M1: Applies the process expanding $(z-\text{"}3\text{"})(z-(\text{their sum})z+\text{their product})$ in order to find $p=...$

A1: $p=-7$ by correct solution only

Question	Scheme	Marks	AOs
2(a)	$\mathbf{r}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}5\\-3\\-4\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}$	M1	1.1b
	$3x-y+2z=10$	A1	2.5
		(2)	
(b)	$\begin{pmatrix}3\\-1\\2\end{pmatrix}\bullet\begin{pmatrix}-1\\-5\\3\end{pmatrix}=8$	B1	1.1b
	$\sqrt{(3)^2+(-1)^2+(2)^2}\,.\sqrt{(-1)^2+(-5)^2+(3)^2}\,\cos\alpha = \text{"}-3+5+6\text{"}$	M1	1.1b
	$\theta = 90^\circ - \arccos\left(\frac{8}{\sqrt{14}.\sqrt{35}}\right)$ or $\sin\theta = \frac{8}{\sqrt{14}.\sqrt{35}}$	M1	2.1
	$\theta = 21.2^\circ$ (1 dp) * cso	A1*	1.1b
		(4)	
(c)	$3(7-\lambda)-(3-5\lambda)+2(-2+3\lambda)=10 \Rightarrow \lambda = ...$	M1	3.1a
	$\lambda = -\frac{1}{2}$	A1	1.1b
	$\overrightarrow{OX}=\begin{pmatrix}7\\3\\-2\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-1\\-5\\3\end{pmatrix}=\begin{pmatrix}...\\...\\...\end{pmatrix}$	M1	1.1b
	$X(7.5, 5.5, -3.5)$	A1ft	1.1b
		(4)	
			(10 marks)

Notes:

(a)

M1: Attempts to apply the formula $\mathbf{r.n} = \mathbf{a.n}$

A1: Correct Cartesian notation. e.g. $3x-y+2z=10$ or $-3x+y-2z=-10$

Note: Do not allow final answer given as $\mathbf{r}\bullet(3\mathbf{i}-\mathbf{j}+2\mathbf{k})=10$, o.e.

(b)

B1: $\overrightarrow{OA}\bullet\mathbf{n}=8$

M1: An attempt to apply the correct dot product formula between **n** and **d**

M1: Depends on previous M mark. Applies the dot product formula to find the angle between Π and l

A1*: 21.2° cso

Question 2 notes continued:

(c)

M1: Substitutes l into Π and solves the resulting equation to give $\lambda = ...$

A1: $\lambda = -\frac{1}{2}$ o.e.

M1: Depends on previous M mark. Substitutes their λ into l and finds at least one of the coordinates

A1ft: $(7.5, 5.5, -3.5)$ but follow through on their value of λ

Question	Scheme	Marks	AOs
3	x = value of savings account, y = value of property bond account, z = value of share dealing account	M1	3.1b
	$x+y+z=5000$ $x+400=y$ $0.015x+0.035y-0.025z=79$ **or** $1.015x+1.035y+0.975z=5079$	A1	1.1b
	Let $\mathbf{A}=\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 0.015 & 0.035 & -0.025\end{pmatrix}$ or $\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 1.015 & 1.035 & 0.975\end{pmatrix}$		
	e.g. $\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 0.015 & 0.035 & -0.025\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}5000\\ -400\\ 79\end{pmatrix}$	M1	3.1a
		A1	1.1b
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 0.015 & 0.035 & -0.025\end{pmatrix}^{-1}\begin{pmatrix}5000\\ -400\\ 79\end{pmatrix}=\begin{pmatrix}...\\ ...\\ ...\end{pmatrix}$	M1	1.1b
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}1800\\ 2200\\ 1000\end{pmatrix}$	A1	1.1b
	Tyler invested £1800 in the savings account, £2200 in the property bond account and £1000 in the share dealing account	A1ft	3.2a
			(7 marks)

Notes:

M1: Attempts to set up 3 equations with 3 unknowns

A1: At least 2 equations are correct with the appropriate variables defined

M1: Sets up a matrix equation of the form, e.g. $\begin{pmatrix}... & ... & ...\\ ... & ... & ...\\ ... & ... & ...\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}...\\ ...\\ ...\end{pmatrix}$, where "…" are numerical values

A1: Correct matrix equation (or equivalent)

M1: Depends on previous M mark. Applies $(\text{their }\mathbf{A})^{-1}\begin{pmatrix}5000\\ \text{their "}-400\text{"}\\ \text{their "79"}\end{pmatrix}$ and obtains at least one value of x, y or z

A1: Correct answer

A1ft: Correct follow through answer in context

Question	Scheme	Marks	AOs
4	$\{w = x-1 \Rightarrow\}\ x = w+1$	B1	3.1a
	$(w+1)^3 + 3(w+1)^2 - 8(w+1) + 6 = 0$	M1	3.1a
	$w^3 + 3w^2 + 3w + 1 + 3(w^2 + 2w+1) - 8w - 8 + 6 = 0$		
	$w^3 + 6w^2 + w + 2 = 0$	M1	1.1b
		A1	1.1b
		A1	1.1b
		(5)	
	Alternative		
	$\alpha + \beta + \gamma = -3,\ \alpha\beta + \beta\gamma + \alpha\gamma = -8,\ \alpha\beta\gamma = -6$	B1	3.1a
	sum roots $= \alpha - 1 + \beta - 1 + \gamma - 1$	M1	3.1a
	$= \alpha + \beta + \gamma - 3 = -3 - 3 = -6$		
	pair sum $= (\alpha-1)(\beta-1) + (\alpha-1)(\gamma-1) + (\beta-1)(\gamma-1)$		
	$= \alpha\beta + \alpha\gamma + \beta\gamma - 2(\alpha+\beta+\gamma) + 3$		
	$= -8 - 2(-3) + 3 = 1$		
	product $= (\alpha-1)(\beta-1)(\gamma-1)$		
	$= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha+\beta+\gamma) - 1$		
	$= -6 - (-8) - 3 - 1 = -2$		
	$w^3 + 6w^2 + w + 2 = 0$	M1	1.1b
		A1	1.1b
		A1	1.1b
		(5)	
	(5 marks)		

Notes:

B1: Selects the method of making a connection between x and w by writing $x = w+1$

M1: Applies the process of substituting their $x = w+1$ into $x^3 + 3x^2 - 8x + 6 = 0$

M1: Depends on previous M mark. Manipulating their equation into the form $w^3 + pw^2 + qw + r = 0$

A1: At least two of p, q, r are correct

A1: Correct final equation

Alternative

B1: Selects the method of giving three correct equations each containing α, β and γ

M1: Applies the process of finding sum roots, pair sum and product

M1: Depends on previous M mark. Applies $w^3 - (\text{their sum roots})\,w^2 + (\text{their pair sum})w - \text{their}\,\alpha\beta\gamma = 0$

A1: At least two of p, q, r are correct

A1: Correct final equation

Question	Scheme	Marks	AOs
5(a)	$\det(\mathbf{M}) = (1)(1) - (\sqrt{3})(-\sqrt{3})$	M1	1.1a
	M is non-singular because $\det(\mathbf{M}) = 4$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	$\text{Area}(S) = 4(5) = 20$	B1ft	1.2
		(1)	
(c)	$k = \sqrt{(1)(1) - (\sqrt{3})(-\sqrt{3})}$	M1	1.1b
	$= 2$	A1ft	1.1b
		(2)	
(d)	$\cos\theta = \frac{1}{2}$ or $\sin\theta = \frac{\sqrt{3}}{2}$ or $\tan\theta = \sqrt{3}$	M1	1.1b
	$\theta = 60°$ or $\frac{\pi}{3}$	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)
M1: An attempt to find $\det(\mathbf{M})$.
A1: $\det(\mathbf{M}) = 4$ **and** reference to zero, e.g. $4 \neq 0$ **and** conclusion.

(b)
B1ft: 20 or a correct ft based on their answer to part (a).

(c)
M1: $\sqrt{(\text{their det}\mathbf{M})}$
A1ft: 2

(d)
M1: **Either** $\cos\theta = \frac{1}{(\text{their } k)}$ **or** $\sin\theta = \frac{\sqrt{3}}{(\text{their } k)}$ **or** $\tan\theta = \sqrt{3}$

A1: $\theta = 60^{\circ}$ or $\frac{\pi}{3}$. Also accept any value satisfying $360n + 60^{\circ}$, $n \in \mathbb{Z}$, o.e.

Question	Scheme	Marks	AOs
6(a)	$n=1,\ \sum_{r=1}^{1} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(1)(2)(3) = 1$	B1	2.2a
	Assume general statement is true for $n=k$ So assume $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$ is true	M1	2.4
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1	2.1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$	A1	1.1b
	$= \frac{1}{6}(k+1)(k+2)(2k+3) = \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$	A1	1.1b
	Then the general result is <u>true for $n=k+1$</u> As the general result has been shown to be <u>true for $n=1$,</u> then the general result <u>is true for all n</u> $\in \mathbb{Z}^+$	A1	2.4
		(6)	
(b)	$\sum_{r=1}^{n} r(r+6)(r-6) = \sum_{r=1}^{n} (r^3 - 36r)$		
	$= \frac{1}{4}n^2(n+1)^2 - \frac{36}{2}n(n+1)$	M1	2.1
		A1	1.1b
	$= \frac{1}{4}n(n+1)\left[n(n+1)-72\right]$	M1	1.1b
	$= \frac{1}{4}n(n+1)(n-8)(n+9)$ * cso	A1*	1.1b
		(4)	
(c)	$\frac{1}{4}n(n+1)(n-8)(n+9) = \frac{17}{6}n(n+1)(2n+1)$	M1	1.1b
	$\frac{1}{4}(n-8)(n+9) = \frac{17}{6}(2n+1)$	M1	1.1b
	$3n^2 - 65n - 250 = 0$	A1	1.1b
	$(3n+10)(n-25) = 0$	M1	1.1b
	(As n must be a positive integer,) $n = 25$	A1	2.3
		(5)	
			(15 marks)

Question 6 notes:

(a)

B1: Checks $n=1$ works for both sides of the general statement

M1: Assumes (general result) true for $n=k$

M1: Attempts to add $(k + 1)^{\text{th}}$ term to the sum of k terms

A1: Correct algebraic work leading to **either** $\frac{1}{6}(k+1)(2k^2+7k+6)$

or $\frac{1}{6}(k+2)(2k^2+5k+3)$ **or** $\frac{1}{6}(2k+3)(k^2+3k+2)$

A1: Correct algebraic work leading to $\frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$

A1: cso leading to a correct induction statement conveying all three underlined points

(b)

M1: Substitutes at least one of the standard formulae into their expanded expression

A1: Correct expression

M1: Depends on previous M mark. Attempt to factorise at least $n(n+1)$ having used

A1*: Obtains $\frac{1}{4}n(n+1)(n-8)(n+9)$ by cso

(c)

M1: Sets their part (a) answer equal to $\frac{17}{6}n(n+1)(2n+1)$

M1: Cancels out $n(n+1)$ from both sides of their equation

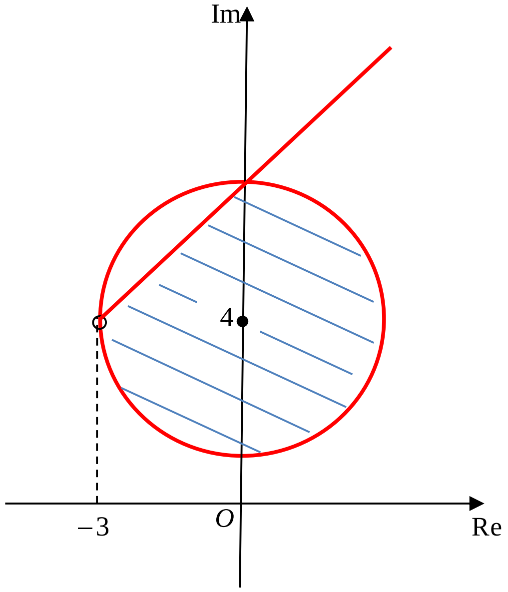
A1: $3n^2 - 65n - 250 = 0$

M1: A valid method for solving a 3 term quadratic equation

A1: Only one solution of $n = 25$

Question	Scheme		Marks	AOs
7(a)	Depth = 0.16 (m)		B1	2.2b
			(1)	
(b)	$y = 1 + kx^2 \Rightarrow 1.16 = 1 + k(0.2)^2 \Rightarrow k = ...$		M1	3.3
	$\Rightarrow k = 4$ cao $\left\{\text{So } y = 1 + 4x^2\right\}$		A1	1.1b
			(2)	
(c)	$\frac{\pi}{4}\int (y-1)\mathrm{d}y$	$\frac{\pi}{4}\int y\,\mathrm{d}y$	B1ft	1.1a
	$= \left\{\frac{\pi}{4}\right\}\int_1^{1.16} (y-1)\mathrm{d}y$	$= \left\{\frac{\pi}{4}\right\}\int_0^{0.16} y\mathrm{d}y$	M1	3.3
	$= \left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2} - y\right]_1^{1.16}$	$= \left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2}\right]_0^{0.16}$	M1 A1	1.1b 1.1b
	$= \frac{\pi}{4}\left(\left(\frac{1.16^2}{2} - 1.16\right) - \left(\frac{1}{2} - 1\right)\right) \left\{= 0.0032\pi\right\}$	$= \frac{\pi}{4}\left(\left(\frac{0.16^2}{2}\right) - (0)\right) \left\{= 0.0032\pi\right\}$		
	$V_{\text{cylinder}} = \pi(0.2)^2(1.16) \left\{= 0.0464\pi\right\}$		B1	1.1b
	$\text{Volume } = 0.0464\pi - 0.0032\pi \left\{= 0.0432\pi\right\}$		M1	3.4
	$= 0.1357168026... = 0.136(\text{m}^3)\ (3\text{sf})$		A1	1.1b
			(7)	
(d)	Any one of e.g. the measurements may not be accurate the inside surface of the bowl may not be smooth there may be wastage of concrete when making the bird bath		B1	3.5b
			(1)	
(e)	Some comment consistent with their values. We do need a reason e.g. $\left[\left(\frac{0.136 - 0.127}{0.127}\right) \times 100 = 7.0866...\right]$ so not a good estimate because the volume of concrete needed to make the bird bath is approximately 7% lower than that predicted by the model **or** We might expect the actual amount of concrete to exceed that which the model predicts due to wastage, so the model does not look suitable since it predicts more concrete than was used		B1ft	3.5a
			(1)	
			(12 marks)	

Question 7 notes:
(a) **B1:** Infers that the maximum depth of the bird bath could be 0.16 (m)
(b) **M1:** Substitutes $y = 1.16$ and $x = 0.2$ or $x = -0.2$ into $y = 1 + kx^2$ and rearranges to give $k = ...$ **A1:** $k = 4$ cao
(c) **B1ft:** Uses the model to obtain either $\frac{\pi}{(\text{their } k)}\int (y-1)\,\mathrm{d}y$ **or** $\frac{\pi}{(\text{their } k)}\int y\,\mathrm{d}y$ **M1:** Chooses limits that are appropriate to their model **M1:** Integrates y (with respect to y) to give $\pm\lambda y^2$, where $\lambda \neq 0$ is a constant **A1:** Uses their model correctly to give either $y - 1 \to \frac{y^2}{2} - y$ or $y \to \frac{y^2}{2}$ **B1:** $V_{\text{cylinder}} = \pi(0.2)^2(1.16)$ or 0.0464π **or** $\frac{29}{625}\pi$, o.e. **M1:** Depends on **both** previous M marks Uses the model to find $V_{\text{their cylinder}} -$ their integrated volume **A1:** 0.136 cao
(d) **B1:** States an acceptable limitation of the model
(e) **B1ft:** Compares the actual volume with their answer to (c). Makes an assessment of the model. E.g. evaluates the percentage error and uses this to make a sensible comment about the model with a reason

Question	Scheme	Marks	AOs
8(a)		M1	1.1b
		A1	1.1b
		M1	1.1b
		A1	2.2a
		M1	3.1a
		A1	1.1b
		(6)	
(b)	$(\arg w)_{\max} = \frac{\pi}{2} + \arcsin\left(\frac{3}{4}\right)$	M1	3.1a
	$= 2.42\ (2\,\text{dp})$ **cao**	A1	1.1b
		(2)	
			(8 marks)

Notes:

(a)

M1: Circle

A1: Centre (0, 4) and above the real axis

M1: Half-line

A1: $(-3, 4)$ positioned correctly and the half-line intersects the top of the circle on the *y*-axis

M1: Depends on **both** previous M marks Shades in a region inside the circle and below the half-line

A1: cso

Note: Final A1 mark is dependent on all previous marks being scored in part (a)

(b)

M1: Uses trigonometry to give an expression for an angle in the range $\left(\frac{\pi}{2}, \pi\right)$ or $(\,90^{\circ}, 180^{\circ}\,)$

A1: 2.42 cao

Question	Scheme	Marks	AOs
9(a)	$\overrightarrow{AB} = \begin{pmatrix}9\\4\\11\end{pmatrix} - \begin{pmatrix}-3\\1\\-7\end{pmatrix} \left\{= \begin{pmatrix}12\\3\\18\end{pmatrix}\right\}$ or $\mathbf{d} = \begin{pmatrix}4\\1\\6\end{pmatrix}$	M1	3.1a
	$\left\{\overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix}-3\\1\\-7\end{pmatrix} + \lambda \begin{pmatrix}12\\3\\18\end{pmatrix}$	M1	1.1b
	$\left\{\overrightarrow{OF} \bullet \overrightarrow{AB} = 0 \Rightarrow \right\} \begin{pmatrix}-3+12\lambda\\1+3\lambda\\-7+18\lambda\end{pmatrix} \bullet \begin{pmatrix}12\\3\\18\end{pmatrix} = 0$ $\Rightarrow -36 + 144\lambda + 3 + 9\lambda - 126 + 324\lambda = 0 \Rightarrow 477\lambda - 159 = 0$	dM1	1.1b
	$\Rightarrow \lambda = \frac{1}{3}$	A1	1.1b
	$\left\{\overrightarrow{OF} = \right\} \begin{pmatrix}-3\\1\\-7\end{pmatrix} + \frac{1}{3}\begin{pmatrix}12\\3\\18\end{pmatrix} = \begin{pmatrix}1\\2\\-1\end{pmatrix}$ and minimum distance $= \sqrt{(1)^2 + (2)^2 + (-1)^2}$	dM1	3.1a
	$= \sqrt{6}$ **or** $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

Question	Scheme	Marks	
	9(a) Alternative 1		
	$\overrightarrow{AB} = \begin{pmatrix}9\\4\\11\end{pmatrix} - \begin{pmatrix}-3\\1\\-7\end{pmatrix} \left\{= \begin{pmatrix}12\\3\\18\end{pmatrix}\right\}$ or $\mathbf{d} = \begin{pmatrix}4\\1\\6\end{pmatrix}$	M1	3.1a
	$\left\{\overrightarrow{OA} = \begin{pmatrix}-3\\1\\-7\end{pmatrix} \text{ and } \overrightarrow{AB} = \begin{pmatrix}12\\3\\18\end{pmatrix} \Rightarrow\right\} \begin{pmatrix}-3\\1\\-7\end{pmatrix} \bullet \begin{pmatrix}12\\3\\18\end{pmatrix}$	M1	1.1b
	$\cos\theta \left\{= \frac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{\lvert\overrightarrow{OA}\rvert . \lvert\overrightarrow{AB}\rvert}\right\} = \frac{\pm\left(\begin{pmatrix}-3\\1\\-7\end{pmatrix} \bullet \begin{pmatrix}12\\3\\18\end{pmatrix}\right)}{\sqrt{(-3)^2+(1)^2+(-7)^2}.\sqrt{(12)^2+(3)^2+(18)^2}}$	dM1	1.1b
	$\left\{\cos\theta = \frac{-36+3-126}{\sqrt{59}.\sqrt{477}} = \frac{-159}{\sqrt{59}.\sqrt{477}}\right\}$		
	$\theta = 161.4038029...$ or $18.59619709...$ or $\sin\theta = 0.3188964021...$	A1	1.1b
	minimum distance $= \sqrt{(-3)^2+(1)^2+(-7)^2}\sin(18.59619709...)$	dM1	3.1a
	$= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	
	9(a) Alternative 2		
	$\overrightarrow{AB} = \begin{pmatrix}9\\4\\11\end{pmatrix} - \begin{pmatrix}-3\\1\\-7\end{pmatrix} \left\{= \begin{pmatrix}12\\3\\18\end{pmatrix}\right\}$ or $\mathbf{d} = \begin{pmatrix}4\\1\\6\end{pmatrix}$	M1	3.1a
	$\left\{\overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix}-3\\1\\-7\end{pmatrix} + \lambda\begin{pmatrix}12\\3\\18\end{pmatrix}$	M1	1.1b
	$\lvert\overrightarrow{OF}\rvert^2 = (-3+12\lambda)^2 + (1+3\lambda)^2 + (-7+18\lambda)^2$	dM1	1.1b
	$= 9 - 72\lambda + 144\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 49 - 252\lambda + 324\lambda^2$		
	$= 477\lambda^2 - 318\lambda + 59$	A1	1.1b
	$= 53(3\lambda - 1)^2 + 6$	dM1	3.1a
	minimum distance $= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

Question	Scheme	Marks	AOs
9(b)	e.g. Fish *F* may not swim in an exact straight line from *A* to *B* Fish *F* may hit an obstacle whilst swimming from *A* to *B* Fish *F* may deviate his path to avoid being caught by the octopus	B1	3.5b
		(1)	
(c)	e.g. Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed Octopus may during the fish *F*'s motion move away from its fixed location at *O*	B1	3.5b
		(1)	
			(9 marks)

Question 9 notes:

(a)

M1: Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OA}-\overrightarrow{OB}$ **or** the direction vector **d**

M1: Applies $\overrightarrow{OA}+\lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent

M1: Depends on previous M mark. Writes down
$(\text{their } \overrightarrow{OF} \text{ which is in terms of } \lambda)\bullet(\text{their } \overrightarrow{AB})=0.$ Can be implied

A1: Lambda is correct. e.g. $\lambda=\frac{1}{3}$ for $\overrightarrow{AB}=\begin{pmatrix}12\\3\\18\end{pmatrix}$ or $\lambda=1$ for $\mathbf{d}=\begin{pmatrix}4\\1\\6\end{pmatrix}$

M1: Depends on previous M mark. Complete method for finding $\left|\overrightarrow{OF}\right|$

A1: $\sqrt{6}$ **or** awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

Alternative 1

(a)

M1: Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OA}-\overrightarrow{OB}$ **or** the direction vector **d**

M1: Realisation that the dot product is required between $\overrightarrow{OA}$ and their $\overrightarrow{AB}.$ (o.e.)

M1: Depends on previous M mark. Applies dot product formula between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$ (o.e.)

A1: $\theta=$ awrt 161.4 or awrt 18.6 or $\sin\theta=$ awrt 0.319

M1: Depends on previous M mark. $(\text{their } OA)\sin(\text{their } \theta)$

A1: $\sqrt{6}$ or awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

Question 9 notes continued:

Alternative 2

(a)

M1: Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OA}-\overrightarrow{OB}$ **or** the direction vector **d**

M1: Applies $\overrightarrow{OA}+\lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent

M1: Depends on previous M mark. Applies Pythagoras by finding $\left|\overrightarrow{OF}\right|^2$, o.e.

A1: $\left|\overrightarrow{OF}\right|^2 = 477\lambda^2 - 318\lambda + 59$

M1: Depends on previous M mark. Method of completing the square or differentiating their $\left|\overrightarrow{OF}\right|^2$ w.r.t. λ

A1: $\sqrt{6}$ or awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

(b)

B1: An acceptable criticism for fish F, which is in context with the question

(c)

B1: An acceptable criticism for the octopus, which is in context with the question

Write your name here

Surname	Other names

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Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2A: Further Pure Mathematics 1 and Further Pure Mathematics 2

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2A

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

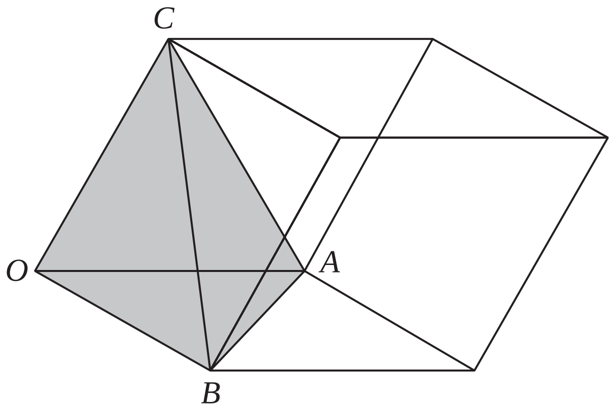


Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are *O*(0, 0, 0), *A*(2, 0, 0), *B*(0, 3, 1) and *C*(1, 1, 2), where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

Question 4 continued

(Total for Question 4 is 10 marks)

5.

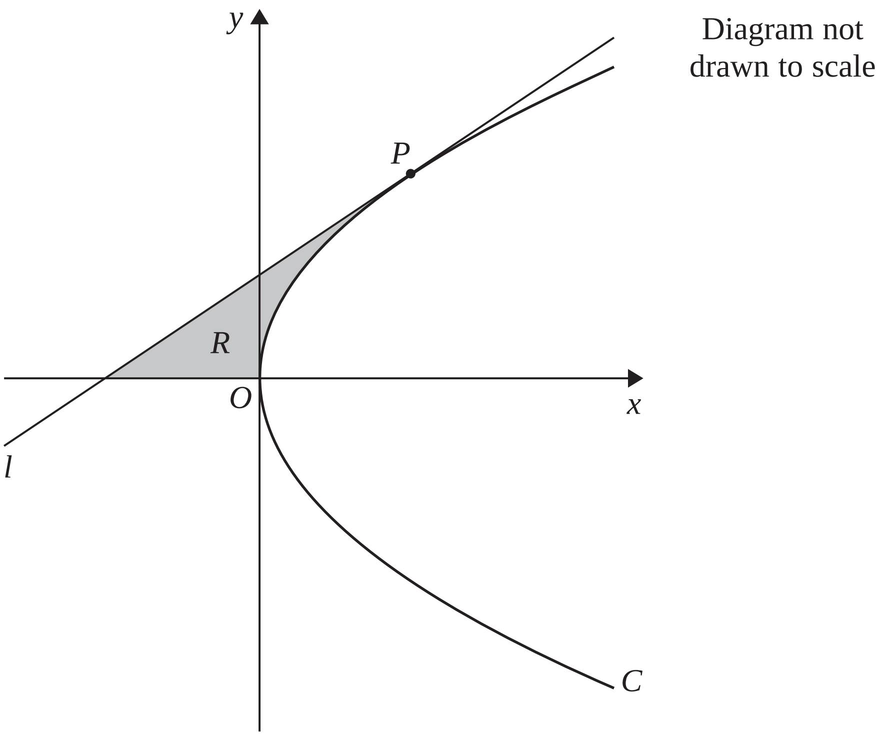


Figure 2

$$\left[\text{You may quote without proof that for the general parabola } y^2 = 4ax,\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{y}\right]$$

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

6. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix}$$

(a) find the characteristic equation of the matrix **A**.

(2)

(b) Hence show that $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$.

(3)

Question 6 continued

(Total for Question 6 is 5 marks)

7. (i) Without performing any division, explain why 8184 is divisible by 6

(2)

(ii) Use the Euclidean algorithm to find integers *a* and *b* such that

$$27a + 31b = 1$$

(4)

Question 7 continued

(Total for Question 7 is 6 marks)

8. A curve C is described by the equation

$$|z - 9 + 12\text{i}| = 2|z|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Sketch C on an Argand diagram.

(2)

Given that w lies on C,

(c) find the largest value of a and the smallest value of b that must satisfy

$$a \leqslant \text{Re}(w) \leqslant b$$

(2)

Question 8 continued

(Total for Question 8 is 8 marks)

9. The operation * is defined on the set $S = \{0, 2, 3, 4, 5, 6\}$ by $x*y = x + y = xy \pmod 7$

*	0	2	3	4	5	6
0						
2		0				
3						5
4						
5		4				
6						

(a) (i) Complete the Cayley table shown above

(ii) Show that S is a group under the operation *

(You may assume the associative law is satisfied.)

(6)

(b) Show that the element 4 has order 3

(2)

(c) Find an element which generates the group and express each of the elements in terms of this generator.

(3)

Question 9 continued

(Total for Question 9 is 11 marks)

10. A population of deer on a large estate is assumed to increase by 10% during each year due to natural causes.

The population is controlled by removing a constant number, Q, of the deer from the estate at the end of each year.

At the start of the first year there are 5000 deer on the estate.

Let P_n be the population of deer at the end of year n.

(a) Explain, in the context of the problem, the reason that the deer population is modelled by the recurrence relation

$$P_n = 1.1P_{n-1} - Q, \qquad P_0 = 5000, \qquad n \in \mathbb{Z}^+$$

(3)

(b) Prove by induction that $P_n = (1.1)^n\,(5000 - 10Q) + 10Q, \qquad n \geqslant 0$

(5)

(c) Explain how the long term behaviour of this population varies for different values of Q.

(2)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Paper 2 Option A

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	$\sec x - \tan x = \dfrac{1}{\dfrac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes:

(a)

M1: Uses $\sec x = \dfrac{1}{\cos x}$ and the *t*-substitutions for both cos *x* and tan *x* to obtain an expression in terms of *t*

M1: Sorts out the sec *x* term, and puts over a common denominator of $1-t^2$

A1*: Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer

(b)

M1: Uses the *t*-substitution for sin *x* in both numerator and denominator

M1: Multiples through by $1+t^2$ in numerator and denominator

A1*: Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result

Question	Scheme	Marks	AOs
2	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step h required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
			(6 marks)

Notes:

B1: Identifies the correct initial conditions and requirement for *h*

M1: Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0

M1: Applies the approximation formula with their values

A1ft: 3.5 or exact equivalent. Follow through their step value

M1: Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5

A1: Applies the approximation and interprets the result to give £396

Question	Scheme	Marks	AOs
3	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R}: x < -2\} \cup \{x \in \mathbb{R}: -1 < x < 0\} \cup \{x \in \mathbb{R}: x > 2\}$	M1 A1	2.2a 2.5
		(6)	
		(6 marks)	

Notes:

M1: Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side

M1: Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors

A1: At least 2 correct critical values found

A1: Exactly 4 correct critical values

M1: Deduces that the 2 "outsides" and the "middle interval" are required. May be by sketch, number line or any other means

A1: Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R}: x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$

Question	Scheme	Marks	AOs
4(a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\mathrm{m}^2)$	A1	1.1b
		(4)	
	Alternative		
	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\mathrm{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=\ldots$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\mathrm{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
	4(b) Alternative		
	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA}=2\mathbf{i}$ and $\mathbf{OB}=3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB=\frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert=\frac{1}{2}\times2\times\sqrt{10}=\sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k})=\mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})\Rightarrow p=\frac{1}{2}$ and so height of tetrahedron is $h=\frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert=\frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V = 5 \times$ Volume of tetrahedron $=5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$=\frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete Measurements in m may not be accurate The surface of the concrete tetrahedron may not be smooth Pockets of air may form when the concrete is being poured	B1	3.2b
		(1)	
		(10 marks)	

Question 4 notes:

Accept use of column vectors throughout

(a)

M1: Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*

M1: Any correct method for the triangle area is fine

M1: Finds $\mathbf{AB}$ and $\mathbf{AC}$ or any other appropriate pair of vectors to use in the vector product and attempts to use them

A1: Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent

(a) **Alternative**

M1: Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides

M1: May use different sides to those shown

M1: Correct full method to find the area of the triangle using their two sides

A1: $\frac{1}{2}\sqrt{35}$ or exact equivalent

Question 4 notes continued:
(b) **M1:** Attempts volume of concrete by finding volume of tetrahedron with appropriate method **M1:** Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted **A1:** Correct value for the volume of concrete **M1:** Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped **A1:** $\frac{25}{3}$ only, ignore reference to units
(b) **Alternative** **M1:** Notes (or works out using scalar products) that $-\mathbf{j}+3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$ **A1:** Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$ **M1:** Solves $\mathbf{i}+\mathbf{j}+2\mathbf{k} - p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})$ to get the height of the tetrahedron $\left[(\mu=\lambda=)\ p=\frac{1}{2},\text{ so } h=\frac{1}{2}\left\|-\mathbf{j}+3\mathbf{k}\right\|=\frac{1}{2}\sqrt{10}\right]$ **M1:** Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference) **A1:** $\frac{25}{3}$ only, ignore reference to units
(c) **B1:** Any acceptable reason in context

Question	Scheme	Marks	AOs
5(a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow$ $P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 16$ so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{y}$	M1	2.2a
	$l:\ y - 8p = \left(\frac{8}{8p}\right)\left(x - 4p^2\right)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow$ Area$(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}\mathrm{d}x$	M1	2.1
	$\int 4x^{\frac{1}{2}}\mathrm{d}x = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\ (+c)$ or $\frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1	1.1b
		A1	1.1b
	Area$(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
	5(c) Alternative 1		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right) \mathrm{d}y$	M1	2.1
	$\int \left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right) \mathrm{d}y = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	
	5(c) Alternative 2		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts px-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l : y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) \mathrm{d}x$	M1	2.1
	$\int \left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right) \mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} \ (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Question 5 notes:

(a)

B1: Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_p = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that *P* lies on *C*

(b)

M1: Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it

M1: Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient *m*, which is in terms of *p*. Accept use of $8p = m(4p^2) + c$ with a clear attempt to find *c*

A1*: Obtains $py = x + 4p^2$ by **cso**

(c)

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$

A1: Finds that *l* cuts the *x*-axis at $x = -9$

M1: Fully correct method for finding the area of *R*

i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\mathrm{d}x$

M1: Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$

A1: Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

(c) Alternative 1

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$

M1: Finds *l* as $x = \frac{3}{2}y - 9$

A1: Fully correct method for finding the area of *R*

M1: i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)\mathrm{d}y$

M1: Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$,where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Question 5 notes continued:

(c) **Alternative 2**

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$

A1: Finds that *l* cuts the *x*-axis at $x = -9$

M1: Fully correct method for finding the area of *R*

i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their}\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}y$

M1: Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$,where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Further Pure Mathematics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6(a)	Consider $\det\begin{pmatrix}3-\lambda & 1\\ 6 & 4-\lambda\end{pmatrix}=(3-\lambda)(4-\lambda)-6$	M1	1.1b
	So $\lambda^2-7\lambda+6=0$ is characteristic equation	A1	1.1b
		(2)	
	So $\mathbf{A}^2=7\mathbf{A}-6\mathbf{I}$	B1ft	1.1b
(b)	Multiplies both sides of their equation by **A** so $\mathbf{A}^3=7\mathbf{A}^2-6\mathbf{A}$	M1	3.1a
	Uses $\mathbf{A}^3=7(7\mathbf{A}-6\mathbf{I})-6\mathbf{A}$ So $\mathbf{A}^3=43\mathbf{A}-42\mathbf{I}$ *	A1*cso	1.1b
		(3)	
	(5 marks)		

Notes:

(a)

M1: Complete method to find characteristic equation

A1: Obtains a correct three term quadratic equation – may use variable other than λ

(b)

B1ft: Uses Cayley Hamilton Theorem to produce equation replacing λ with **A** and constant term with constant multiple of identity matrix, **I**

M1: Multiplies equation by **A**

A1*: Replaces $\mathbf{A}^2$ by linear expression in **A** and achieves printed answer with no errors

Question	Scheme	Marks	AOs
7(i)	Adding digits 8 + 1 + 8 + 4 = 21 which is divisible by 3 (or continues to add digits giving 2+1=3 which is divisible by 3) so concludes that 8184 is divisible by 3	M1	1.1b
	8184 is even, so is divisible by 2 and as divisible by both 3 and 2, so it is divisible by 6	A1	1.1b
		(2)	
(ii)	Starts Euclidean algorithm $31=27\times1+4$ and $27 = 4 \times 6 + 3$	M1	1.2
	$4 = 3 \times 1 + 1$ (so hcf = 1)	A1	1.1b
	So $1 = 4 – 3 \times 1 = 4 – (27 – 4 \times 6) \times 1 = 4 \times 7 – 27 \times 1$	M1	1.1b
	$(31 – 27 \times 1) \times 7 – 27 \times1 = 31 \times 7 – 27 \times 8$ $a = –8$ and $b = 7$	A1cso	1.1b
		(4)	
			(6 marks)

Notes:

(i)

M1: Explains divisibility by 3 rule in context of this number by adding digits

A1: Explains divisibility by 2, giving last digit even as reason and makes conclusion that number is divisible by 6

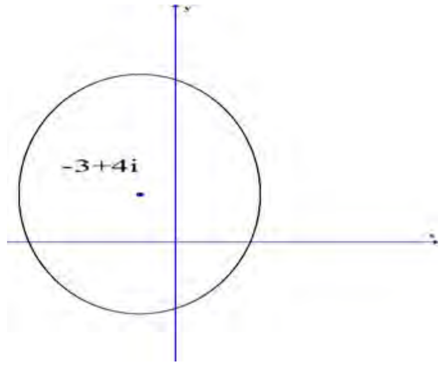
(ii)

M1: Uses Euclidean algorithm showing two stages

A1: Completes the algorithm. Does not need to state that hcf = 1

M1: Starts reversal process, doing two stages and simplifying

A1cso: Correct completion, giving clear answer following complete solution

Question	Scheme	Marks	AOs
8(a)	$(x-9)^2+(y+12)^2 = 4[x^2+y^2]$	M1	2.1
	$3x^2+3y^2+18x-24y-225=0$ which is the equation of a circle	A1*	2.2a
	As $x^2+y^2+6x-8y-75=0$ so $(x+3)^2+(y-4)^2=10^2$	M1	1.1b
	Giving centre at (-3, 4) and radius = 10	A1ft	1.1b
		(4)	
(b)		M1	1.1b
		A1	1.1b
		(2)	
(c)	Values range from **their** – 3 –10 to their –3 + 10	M1	3.1a
	So $-13 \le \text{Re}(w) \le 7$	A1ft	1.1b
		(2)	
			(8 marks)

Notes:

(a)
M1: Obtains an equation in terms of *x* and *y* using the given information
A1: Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle
M1: Completes the square for their equation to find centre and radius
A1ft: Both correct

(b)
M1: Draws a circle with centre and radius as given from **their** equation
A1: Correct circle drawn, as above, with centre at – 3 + 4i and passing through all four quadrants

(c)
M1: Attempts to find where a line parallel to the real axis, passing through the centre of the circle, meets the circle so using " their – 3 –10" to "their –3 + 10"
A1ft: Correctly obtains the correct answer for their centre and radius

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="2">9(a)(i)</td><td><table>
<tr><td>*</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
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<tr><td>*</td><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
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<tr><td>6</td><td>6</td><td>3</td><td>5</td><td>0</td><td>2</td><td>4</td></tr>
</table></td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td rowspan="4">(ii)</td><td>Identity is zero and there is closure as shown above</td><td>M1</td><td>2.1</td></tr>
<tr><td>3 and 5 are inverses, 4 and 6 are inverses, 2 is self-inverse,
0 is identity so is self-inverse</td><td>M1</td><td>2.5</td></tr>
<tr><td>Associative law may be assumed so <i>S</i> forms a group</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(6)</td><td></td></tr>
<tr><td rowspan="3">(b)</td><td>4*4*4 = 4* (4 * 4) = 4 * 6 or 4*4*4 = (4* 4) * 4 = 6* 4</td><td>M1</td><td>2.1</td></tr>
<tr><td>= 0 (the identity) so 4 has order 3</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="3">(c)</td><td>3 and 5 each have order 6 so either generates the group</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Either $3^1 = 3, 3^2 = 4, 3^3 = 2, 3^4 = 6, 3^5 = 5, 3^6 = 0$
Or $5^1 = 5, 5^2 = 6, 5^3 = 2, 5^4 = 4, 5^5 = 3, 5^6 = 0$</td><td>A1, A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(11 marks)</td></tr>
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Question 9 notes:
(a)(i) **M1:** Begins completing the table – obtaining correct first row and first column and using symmetry **M1:** Mostly correct – three rows or three columns correct (so demonstrates understanding of using * **A1:** Completely correct
(a)(ii) **M1:** States closure and identifies the identity as zero **M1:** Finds inverses for each element **A1:** States that associative law is satisfied and so all axioms satisfied and *S* is a group
(b) **M1:** Clearly begins process to find 4*4*4 reaching 6*4 or 4*6 with clear explanation **A1:** Gives answer as zero, states identity and deduces that order is 3
(c) **M1:** Finds either 3 or 5 or both **A1:** Expresses four of the six terms as powers of either generator correctly (may omit identity and generator itself) **A1:** Expresses all six terms correctly in terms of either 3 or 5 (Do not need to give both)

Question	Scheme	Marks	AOs
10(a)	P_{n-1} is the population at the end of year *n* – 1 and this is increased by 10% by the end of year *n*, so is multiplied by 110% = 1.1 to give $1.1\times P_{n-1}$ as new population by natural causes	B1	3.3
	Q is subtracted from $1.1\times P_{n-1}$ as *Q* is the number of deer removed from the estate	B1	3.4
	So $P_n = 1.1P_{n-1} - Q, \quad P_0 = 5000$ as population at start is 5000 and $n \in \mathbb{Z}^+$	B1	1.1b
		(3)	
(b)	Let *n* = 0, then $P_0 = (5000-10Q)(1.1)^0 + 10Q = 5000$ so result is true when *n* = 0	B1	2.1
	Assume result is true for *n* = *k* , $P_k = (1.1)^k(5000-10Q)+10Q,$ then **as** $P_{k+1} = 1.1P_k - Q,$ so $P_{k+1} = \ldots$	M1	2.4
	$P_{k+1} = 1.1\times1.1^k(5000-10Q)+1.1\times10Q - Q$	A1	1.1b
	So $P_{k+1} = (5000-10Q)(1.1)^{k+1} + 10Q,$	A1	1.1b
	Implies result holds for *n* = *k* + 1 and so by induction $P_n = (5000-10Q)(1.1)^n + 10Q,$ is true for all integer *n*	B1	2.2a
		(5)	
(c)	For *Q* < 500 the population of deer will grow, for *Q* > 500 the population of deer will fall	B1	3.4
	For *Q* = 500 the population of deer remains steady at 5000,	B1	3.4
		(2)	
		(10 marks)	

Notes:

(a)
B1: Need to see 10% increase linked to multiplication by scale factor 1.1
B1: Needs to explain that subtraction of *Q* indicates the removal of *Q* deer from population
B1: Needs complete explanation with mention of $P_n = 1.1P_{n-1} - Q, \quad P_0 = 5000$ being the initial number of deer

(b)
B1: Begins proof by induction by considering *n* = 0
M1: Assumes result is true for *n* = *k* and uses iterative formula to consider *n* = *k* + 1
A1: Correct algebraic statement
A1: Correct statement for *k* + 1 in required form
B1: Completes the inductive argument

(c)
B1: Consideration of both possible ranges of values for *Q* as listed in the scheme
B1: Gives the condition for the steady state

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2B: Further Pure Mathematics 1 and Further Statistics 1

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2B

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of *x* for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

4.

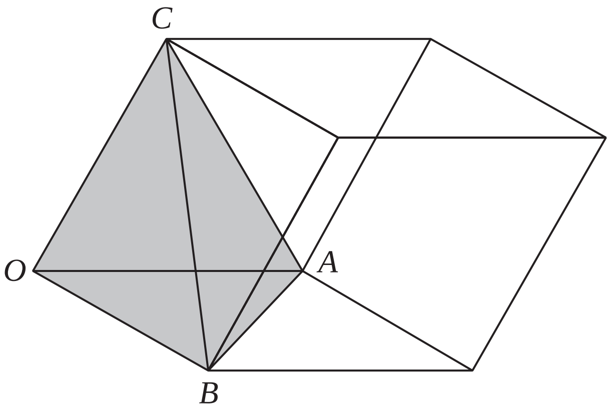

Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

Question 4 continued

(Total for Question 4 is 10 marks)

5.

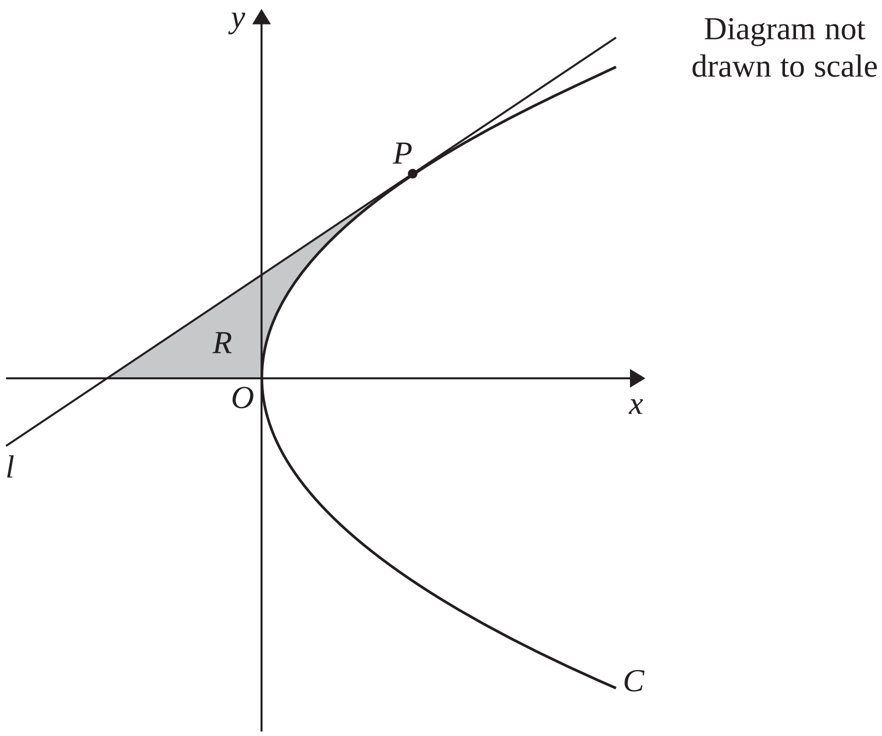

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\dfrac{dy}{dx} = \dfrac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\dfrac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

6. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.

(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6

(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.

(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.

(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.

(1)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. The discrete random variable X has probability distribution given by

x	–1	0	1	2	3
$P(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $E(Y) = -4$ and $P(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $E(Y^2) = 75$

(b) find the exact value of $\text{Var}(X)$

(2)

(c) Find $P(Y > X)$

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

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Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

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Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 11 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Paper 2 Option **B**

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	$\sec x - \tan x = \dfrac{1}{\dfrac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes:

(a)

M1: Uses $\sec x = \dfrac{1}{\cos x}$ and the *t*-substitutions for both cos *x* and tan *x* to obtain an expression in terms of *t*

M1: Sorts out the sec *x* term, and puts over a common denominator of $1-t^2$

A1*: Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer

(b)

M1: Uses the *t*-substitution for sin *x* in both numerator and denominator

M1: Multiples through by $1+t^2$ in numerator and denominator

A1*: Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result

Question	Scheme	Marks	AOs
2	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step h required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
			(6 marks)

Notes:

B1: Identifies the correct initial conditions and requirement for h

M1: Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0

M1: Applies the approximation formula with their values

A1ft: 3.5 or exact equivalent. Follow through their step value

M1: Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5

A1: Applies the approximation and interprets the result to give £396

Question	Scheme	Marks	AOs
3	$\dfrac{1}{x} < \dfrac{x}{x+2}$		
	$\dfrac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\dfrac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \dfrac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1 A1	2.2a 2.5
		(6)	
		(6 marks)	

Notes:

M1: Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side

M1: Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors

A1: At least 2 correct critical values found

A1: Exactly 4 correct critical values

M1: Deduces that the 2 "outsides" and the "middle interval" are required. May be by sketch, number line or any other means

A1: Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$

Question	Scheme	Marks	AOs
4(a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\mathrm{m}^2)$	A1	1.1b
		(4)	
	Alternative		
	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\mathrm{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=\ldots$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\mathrm{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
	4(b) Alternative		
	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB = \frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert = \frac{1}{2}\times 2\times\sqrt{10} = \sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k}) \Rightarrow p = \frac{1}{2}$ and so height of tetrahedron is $h = \frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert = \frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V = 5 \times$ Volume of tetrahedron $= 5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$= \frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete Measurements in m may not be accurate The surface of the concrete tetrahedron may not be smooth Pockets of air may form when the concrete is being poured	B1	3.2b
		(1)	
		(10 marks)	

Question 4 notes:

Accept use of column vectors throughout

(a)

M1: Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*

M1: Any correct method for the triangle area is fine

M1: Finds $\mathbf{AB}$ and $\mathbf{AC}$ or any other appropriate pair of vectors to use in the vector product and attempts to use them

A1: Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent

(a) **Alternative**

M1: Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides

M1: May use different sides to those shown

M1: Correct full method to find the area of the triangle using their two sides

A1: $\frac{1}{2}\sqrt{35}$ or exact equivalent

Question 4 notes continued:

(b)

M1: Attempts volume of concrete by finding volume of tetrahedron with appropriate method

M1: Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted

A1: Correct value for the volume of concrete

M1: Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped

A1: $\frac{25}{3}$ only, ignore reference to units

(b) Alternative

M1: Notes (or works out using scalar products) that $-\mathbf{j}+\mathbf{3k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$

A1: Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$

M1: Solves $\mathbf{i}+\mathbf{j}+\mathbf{2k} - p(-\mathbf{j}+\mathbf{3k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j}+\mathbf{k})$ to get the height of the tetrahedron

$$\left[(\mu = \lambda =)\ p = \frac{1}{2},\ \text{so } h = \frac{1}{2}\left|-\mathbf{j}+3\mathbf{k}\right| = \frac{1}{2}\sqrt{10}\right]$$

M1: Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference)

A1: $\frac{25}{3}$ only, ignore reference to units

(c)

B1: Any acceptable reason in context

Question	Scheme	Marks	AOs
5(a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 16$ so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{y}$	M1	2.2a
	$l:\ y - 8p = \left(\frac{8}{8p}\right)\left(x - 4p^2\right)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}\mathrm{d}x$	M1	2.1
	$\int 4x^{\frac{1}{2}}\mathrm{d}x = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\ (+c)$ or $\frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
	5(c) Alternative 1		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12}\left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right)\mathrm{d}y$	M1	2.1
	$\int\left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right)\mathrm{d}y = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \; (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	
	5(c) Alternative 2		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts px-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9\left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}x$	M1	2.1
	$\int\left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right)\mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} \; (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
			(12 marks)

Question 5 notes:

(a)

B1: Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_p = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that *P* lies on *C*

(b)

M1: Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it

M1: Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient *m*, which is in terms of *p*. Accept use of $8p = m(4p^2) + c$ with a clear attempt to find *c*

A1*: Obtains $py = x + 4p^2$ by **cso**

(c)

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$

A1: Finds that *l* cuts the *x*-axis at $x = -9$

M1: Fully correct method for finding the area of *R*

i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\,\mathrm{d}x$

M1: Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$

A1: Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

(c) Alternative 1

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$

M1: Finds *l* as $x = \frac{3}{2}y - 9$

A1: Fully correct method for finding the area of *R*

M1: i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)\mathrm{d}y$

M1: Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Question 5 notes continued:

(c) Alternative 2

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$

A1: Finds that *l* cuts the *x*-axis at $x = -9$

M1: Fully correct method for finding the area of *R*

i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their } \left(\frac{2}{3}x + 6 \right) - \left(4x^{\frac{1}{2}} \right) \right) \mathrm{d}y$

M1: Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\left(\frac{2}{3}x + 6 \right) - \left(4x^{\frac{1}{2}} \right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Further Statistics 1 Mark Scheme (Section **B**)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="2">6(a)</td><td>H_0: There is no association between language and gender</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="2">(b)</td><td>$\frac{54 \times 85}{150} = 30.6$ *</td><td>B1*cso</td><td>1.1b</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="4">(c)</td><td><table><tr><td colspan="2" rowspan="2">Expected frequencies</td><td colspan="3">Language</td></tr><tr><td>French</td><td>Spanish</td><td>Mandarin</td></tr><tr><td rowspan="2">Gender</td><td>Male</td><td>26.43…</td><td>23.4</td><td>15.16…</td></tr><tr><td>Female</td><td>34.56…</td><td>[30.6]</td><td>19.83…</td></tr></table></td><td>M1</td><td>2.1</td></tr>
<tr><td>$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + ... + \frac{(15-19.83)^2}{19.83}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Awrt 3.6/3.7</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td rowspan="3">(d)</td><td>Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$</td><td>M1</td><td>3.1b</td></tr>
<tr><td>As $\sum \frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="2">(e)</td><td>Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4">(8 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">(a)
B1: For correct hypothesis in context</td></tr>
<tr><td colspan="4">(b)
B1*: For a correct calculation leading to the given answer and no errors seen</td></tr>
<tr><td colspan="4">(c)
M1: For attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1: For applying $\sum \frac{(O-E)^2}{E}$
A1: awrt 3.6 or 3.7</td></tr>
<tr><td colspan="4">(d)
M1: For using degrees of freedom to set up a χ^2 model critical value
A1: For correct comparison and conclusion</td></tr>
<tr><td colspan="4">(e)
A1ft: For correct conclusion with supporting reason</td></tr>
</table>

Question	Scheme	Marks	AOs
7(a)	$-4 = 2 - 5\mathrm{E}(X)$	M1	3.1a
	$\mathrm{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2 \qquad \boxed{1}$		
	$\mathrm{P}(Y \geqslant -3) = 0.45$ gives $\mathrm{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\mathrm{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45 \qquad \boxed{2}$		
	$2a + b + 2c = 1 \qquad \boxed{3}$	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix}\begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. $\boxed{3} - \boxed{2} \Rightarrow b + c = 0.55$ sub. $2(b + c)$ into $\boxed{1} \Rightarrow a = 0.1$ etc		
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\mathrm{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	[$\mathrm{Var}(Y) = 5^2\mathrm{Var}(X)$ implies] $\mathrm{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\mathrm{P}(Y > X) = \mathrm{P}(2 - 5X > X) \rightarrow \mathrm{P}(X < \frac{1}{3})$	M1	3.1a
	$\mathrm{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
			(11 marks)

Notes:

(a)

M1: For using given information to find an expression for $\mathrm{E}(X)$ i.e. use of $\mathrm{E}(Y) = 2 - 5\mathrm{E}(X)$

M1: For use of $\sum x\mathrm{P}(X = x) =$ '1.2'

M1: For use of $\mathrm{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \mathrm{P}(X = x) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

Question **7** notes continued:

Another method for part (a) is:

M1: For using given information to find the probability distribution for Y leading to an expression for E(Y)

M1: For use of $\sum y\mathrm{P}(Y = y) = –4$

M1: For use of P($Y \geqslant\ –3$) = 0.45 to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \mathrm{P}(Y = y) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

(b)

M1: For use of Var(Y) = E(Y^2) – [E(Y)]^2 (may be implied by a correct answer)

A1: For use of Var(aX) = a^2 Var(X) to reach 2.36 or exact equivalent

(c)

M1: For rearranging to the form P($X < k$)

A1ft: 0.1' + '025' (provided their a and c and their $a + c$ are all probabilities)

Another method for part (c) is:

M1: For comparing distribution of X with distribution of Y to identify $X = –1$ and $X = 0$

A1ft: '0.1' + '025' (provided their a and c and their $a + c$ are all probabilities)

Question	Scheme	Marks	AOs
8(a)	$X \sim \mathrm{Po}(2.6)$ $Y \sim \mathrm{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= \mathrm{P}(X=2) \times \mathrm{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **<u>0.0544</u>**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow W \sim \mathrm{Po}(3.8)$	M1	3.4
	$\mathrm{P}(W = 3) = 0.20458\ldots.$ awrt **<u>0.205</u>**	A1	1.1b
		(2)	
(c)	$T \sim \mathrm{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\mathrm{P}(T < 9) = 0.64819\ldots$ awrt **<u>0.648</u>**	A1	1.1b
		(2)	
(d)	**(i)** Mean = np = **<u>2.4</u>**	B1	1.1b
	(ii) Variance = $np(1-p) = 2.3904$ awrt **<u>2.39</u>**	B1	1.1b
		(2)	
(e)	**(i)** [$D \sim \mathrm{Po}(2.4)$ $\mathrm{P}(D \leqslant 4)$] $= 0.9041\ldots$ awrt **<u>0.904</u>**	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes:

(a)
M1: For $\mathrm{P}(X=2) \times \mathrm{P}(Y=2)$ from $X \sim \mathrm{Po}(2.6)$ and $Y \sim \mathrm{Po}(1.2)$ i.e. correct models (may be implied by correct answer)
A1: **awrt 0.0544**

(b)
M1: For combining Poisson distributions and use of Po(‘3.8’) (may be implied by correct answer)
A1: **awrt 0.205**

(c)
M1: For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)
A1: **awrt 0.648**

(d)(i)
B1: For **2.4**

(d)(ii)
B1: For **awrt 2.39**

(e)(i)
B1: For **awrt 0.904**

(e)(ii)
B1: For a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
9(a)	**(i)** P(X = 1) = 0.34523… awrt **0.345**	B1	1.1b
	(ii) P($X \leqslant$ 4) = 0.98575… awrt **0.986**	B1	1.1b
		(2)	
(b)	$\frac{(0\times10)+1\times16+2\times7+3\times4+4\times2+(5\times0)+6\times1}{40}=1.4*$	B1*cso	1.1b
		(1)	
(c)	r = 40 × ‘0.34523…’ s = 40 × ‘1 – 0.986…’	M1	3.4
	r = **13.81** s = **0.57**	A1ft	1.1b
		(2)	
(d)	H$_0$: The Poisson distribution is a suitable model H$_1$: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(10-9.86)^2}{9.86}+...+\frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **1.1**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H$_0$ since 1.10 < $\chi^2_{2,(0.05)}=5.991$). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		(6)	
			(11 marks)

Notes:

(a)(i)
B1: awrt 0.345

(a)(ii)
B1: awrt 0.986

(b)
B1*: For a fully correct calculation leading to given answer with no errors seen

(c)
M1: For attempt at r or s (may be implied by correct answers)
A1ft: For both values correct (follow through their answers to part (a))

(d)
B1: For both hypotheses correct (lambda should not be defined so correct use of the model)
M1: For understanding the need to combine cells before calculating the test statistic (may be implied)
M1: For attempt to find the test statistic using $\chi^2=\sum\frac{(O-E)^2}{E}$
A1: awrt 1.1
B1: For realising that there are 2 degrees of freedom leading to a critical value of $\chi_2^{\,2}(0.05)=5.991$
A1: Concluding that a Poisson model is suitable for the number of mortgages approved each week

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2C: Further Pure Mathematics 1 and Further Mechanics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 40 minutes

Paper Reference

8FM0/2C

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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S 5 8 5 4 0 A 0 1 2 2

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \equiv \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \equiv (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of x for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

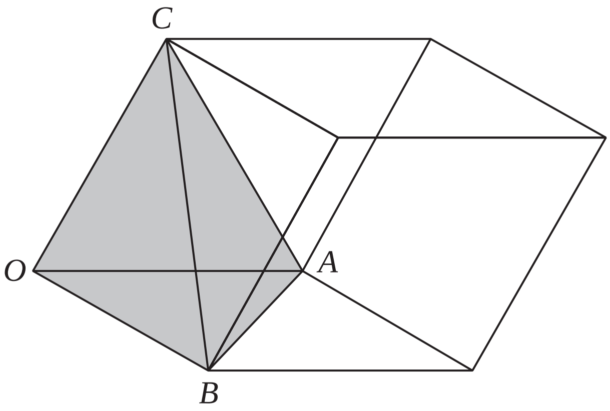

Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.

(4)

(b) Find the volume of glass contained in this parallelepiped.

(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.

(1)

Question 4 continued

(Total for Question 4 is 10 marks)

5.

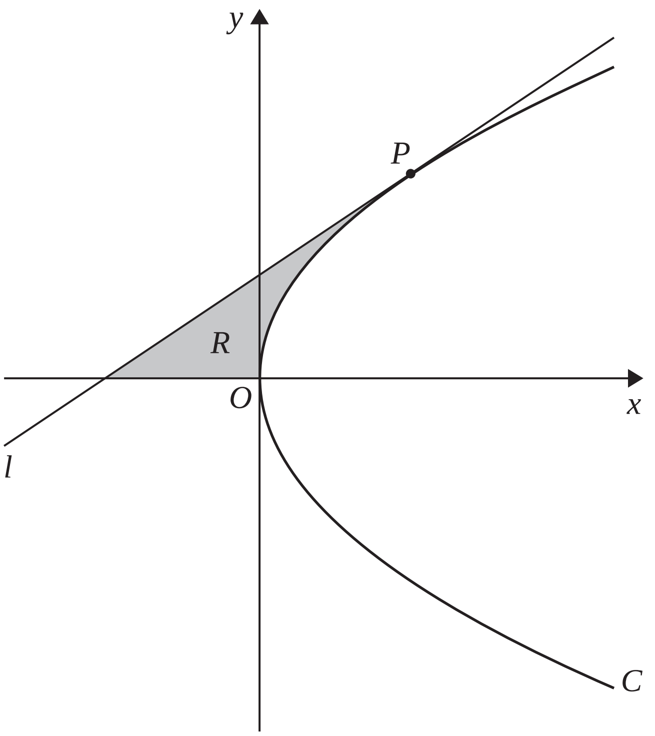

Diagram not
drawn to scale

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\dfrac{dy}{dx} = \dfrac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\dfrac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

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Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

6. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 6 continued

(Total for Question 6 is 10 marks)

7. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 7 continued

(Total for Question 7 is 6 marks)

8. [*In this question use* $g = 10\,\text{m}\,\text{s}^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\,\text{m}\,\text{s}^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\text{m}\,\text{s}^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\text{m}\,\text{s}^{-1}$.

(5)

Question 8 continued

(Total for Question 8 is 8 marks)

9. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \dfrac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \dfrac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

Question 9 continued

(Total for Question 9 is 16 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Paper 2 Option C

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	$\sec x - \tan x = \dfrac{1}{\dfrac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes:

(a)

M1: Uses $\sec x = \dfrac{1}{\cos x}$ and the *t*-substitutions for both cos *x* and tan *x* to obtain an expression in terms of *t*

M1: Sorts out the sec *x* term, and puts over a common denominator of $1-t^2$

A1*: Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer

(b)

M1: Uses the *t*-substitution for sin *x* in both numerator and denominator

M1: Multiples through by $1+t^2$ in numerator and denominator

A1*: Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result

Question	Scheme	Marks	AOs
2	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step h required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
		(6 marks)	

Notes:

B1: Identifies the correct initial conditions and requirement for h

M1: Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0

M1: Applies the approximation formula with their values

A1ft: 3.5 or exact equivalent. Follow through their step value

M1: Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5

A1: Applies the approximation and interprets the result to give £396

Question	Scheme	Marks	AOs
3	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1 A1	2.2a 2.5
		(6)	
		(6 marks)	

Notes:

M1: Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side

M1: Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors

A1: At least 2 correct critical values found

A1: Exactly 4 correct critical values

M1: Deduces that the 2 "outsides" and the "middle interval" are required. May be by sketch, number line or any other means

A1: Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$

Question	Scheme	Marks	AOs
4(a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\mathrm{m}^2)$	A1	1.1b
		(4)	
	Alternative		
	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\mathrm{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=\ldots$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\mathrm{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
	4(b) Alternative		
	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB = \frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert = \frac{1}{2}\times 2\times\sqrt{10} = \sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k}) \Rightarrow p=\frac{1}{2}$ and so height of tetrahedron is $h=\frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert = \frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V = 5$ × Volume of tetrahedron $= 5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$= \frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete Measurements in m may not be accurate The surface of the concrete tetrahedron may not be smooth Pockets of air may form when the concrete is being poured	B1	3.2b
		(1)	

(10 marks)

Question 4 notes:

Accept use of column vectors throughout

(a)

M1: Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*

M1: Any correct method for the triangle area is fine

M1: Finds $\mathbf{AB}$ and $\mathbf{AC}$ or any other appropriate pair of vectors to use in the vector product and attempts to use them

A1: Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent

(a) Alternative

M1: Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides

M1: May use different sides to those shown

M1: Correct full method to find the area of the triangle using their two sides

A1: $\frac{1}{2}\sqrt{35}$ or exact equivalent

Question 4 notes continued:

(b)

M1: Attempts volume of concrete by finding volume of tetrahedron with appropriate method

M1: Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted

A1: Correct value for the volume of concrete

M1: Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped

A1: $\frac{25}{3}$ only, ignore reference to units

(b) **Alternative**

M1: Notes (or works out using scalar products) that $-\mathbf{j}+\mathbf{3k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$

A1: Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$

M1: Solves $\mathbf{i}+\mathbf{j}+\mathbf{2k} - p(-\mathbf{j}+\mathbf{3k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j}+\mathbf{k})$ to get the height of the tetrahedron $\left[(\mu = \lambda =)\ p = \frac{1}{2},\ \text{so } h = \frac{1}{2}\left|-\mathbf{j}+3\mathbf{k}\right| = \frac{1}{2}\sqrt{10}\right]$

M1: Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference)

A1: $\frac{25}{3}$ only, ignore reference to units

(c)

B1: Any acceptable reason in context

Question	Scheme	Marks	AOs
5(a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 16$ so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{y}$	M1	2.2a
	$l:\ y - 8p = \left(\frac{8}{8p}\right)\left(x - 4p^2\right)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \ldots$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \ldots$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow$ Area$(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}\mathrm{d}x$	M1	2.1
	$\int 4x^{\frac{1}{2}}\mathrm{d}x = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\ (+c)$ or $\frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1	1.1b
		A1	1.1b
	Area$(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
	5(c) Alternative 1		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right) \mathrm{d}y$	M1	2.1
	$\int \left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right) \mathrm{d}y = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	
	5(c) Alternative 2		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts px-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l : y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) \mathrm{d}x$	M1	2.1
	$\int \left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right) \mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
			(12 marks)

Question 5 notes:

(a)

B1: Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_p = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that *P* lies on *C*

(b)

M1: Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it

M1: Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient *m*, which is in terms of *p*. Accept use of $8p = m(4p^2) + c$ with a clear attempt to find *c*

A1*: Obtains $py = x + 4p^2$ by **cso**

(c)

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$

A1: Finds that *l* cuts the *x*-axis at $x = -9$

M1: Fully correct method for finding the area of *R*

i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\,\mathrm{d}x$

M1: Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$

A1: Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

(c) Alternative 1

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$

M1: Finds *l* as $x = \frac{3}{2}y - 9$

A1: Fully correct method for finding the area of *R*

M1: i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)\mathrm{d}y$

M1: Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Question 5 notes continued:

(c) **Alternative 2**

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$

A1: Finds that *l* cuts the *x*-axis at $x = -9$

M1: Fully correct method for finding the area of *R*

i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their} \left(\frac{2}{3}x + 6 \right) - \left(4x^{\frac{1}{2}} \right) \right) \mathrm{d}y$

M1: Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$,where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\left(\frac{2}{3}x + 6 \right) - \left(4x^{\frac{1}{2}} \right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Further Mechanics 1 Mark Scheme (Section **B**)

Question	Scheme	Marks	AOs
6(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g \quad => \; v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g \times 0.6 \; => \; u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1*	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh => \; h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		(1)	
			(10 marks)

Notes:

(a)
M1: For a complete method to find v
A1: For a correct value (may be numerical)
M1: For a complete method to find u
A1: For a correct value (may be numerical)
M1: For finding both v and u and use of Newton's Law of Restitution
A1*: For the given answer

(b)
M1: For use of Newton's Law of Restitution to find rebound speed
M1: For a complete method to find h
A1: For 0.15 (m) oe

(c)
B1: For a clear description including reference to a quarter

Question	Scheme	Marks	AOs
7(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5\, g \times 20$	A1	1.1b
	= 58.25 = 58 (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, 20 R = 58.25	M1	3.3
	R = 2.9125 = 2.9 or 2.91	A1ft	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)
M1: For a difference in KE and PE
A1: For a correct expression
A1: For either 58 (2sf) or 58.3(3sf)

(b)
M1: For use of work-energy principle
A1ft: For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

(c)
B1: For variable resistance oe

Question	Scheme	Marks	AOs
8(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force × Speed = 30 × 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force × Speed = 70 × 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)
B1: For force = 30 seen
M1: For use of $P = Fv$
A1ft: For 120 (W), follow through on their '30'

(b)
M1: For resolving parallel to the slope with correct no. of terms and 60g resolved
A1: For a correct equation
A1: For $F = 70$
M1: For use of $P = Fv$
A1ft: For 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
9(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
		(16 marks)	

Notes:

(a)
M1: For use of CLM, with correct no. of terms, condone sign errors
A1: For a correct equation
M1: For use of Newton’s Law of Restitution, with e on the correct side
A1: For a correct equation
M1: For setting up *two* equations and solving their equations for v
A1: For a correct expression for v

(b)
M1: For use of an appropriate inequality
A1: For a complete range of values of e

(c)
M1: For solving their equations for w
A1: For the given answer

Question **9** notes continued:

(d)

M1: For substituting $e = \frac{5}{9}$ into their v and w

A1: For correct expressions for v and w

M1: For use of Newton's Law of Restitution, with e on the correct side

M1: For use of appropriate inequality

A1: For a correct inequality

A1: For a correct range

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2D: Further Pure Mathematics 1 and Decision Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 40 minutes

Paper Reference

8FM0/2D

You must have:

Decision Mathematics question insert

Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\sec x - \tan x \ldots \frac{1-t}{1+t} \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

(b) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ and the answer to part (a) to prove that

$$\frac{1-\sin x}{1+\sin x} \ldots (\sec x - \tan x)^2 \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. The value, V hundred pounds, of a particular stock t hours after the opening of trading on a given day is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V^2 - t}{t^2 + tV} \qquad 0 < t < 8.5$$

A trader purchases £300 of the stock one hour after the opening of trading.

Use two iterations of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$ to estimate, to the nearest £, the value of the trader's stock half an hour after it was purchased.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Use algebra to find the set of values of *x* for which

$$\frac{1}{x} < \frac{x}{x+2}$$

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

4.

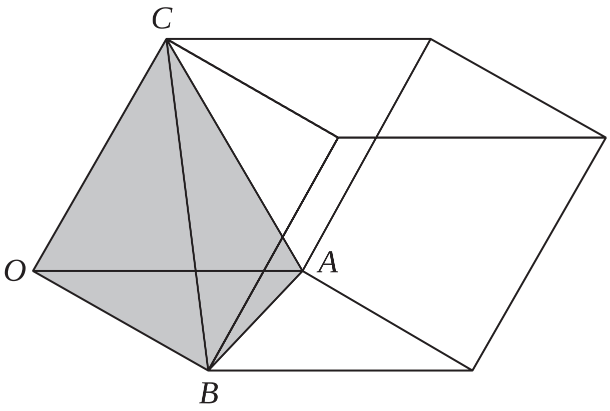

Figure 1

Figure 1 shows a sketch of a solid sculpture made of glass and concrete. The sculpture is modelled as a parallelepiped.

The sculpture is made up of a concrete solid in the shape of a tetrahedron, shown shaded in Figure 1, whose vertices are $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 1)$ and $C(1, 1, 2)$, where the units are in metres. The rest of the solid parallelepiped is made of glass which is glued to the concrete tetrahedron.

(a) Find the surface area of the glued face of the tetrahedron.
(4)

(b) Find the volume of glass contained in this parallelepiped.
(5)

(c) Give a reason why the volume of concrete predicted by this model may not be an accurate value for the volume of concrete that was used to make the sculpture.
(1)

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Question 4 continued

(Total for Question 4 is 10 marks)

5.

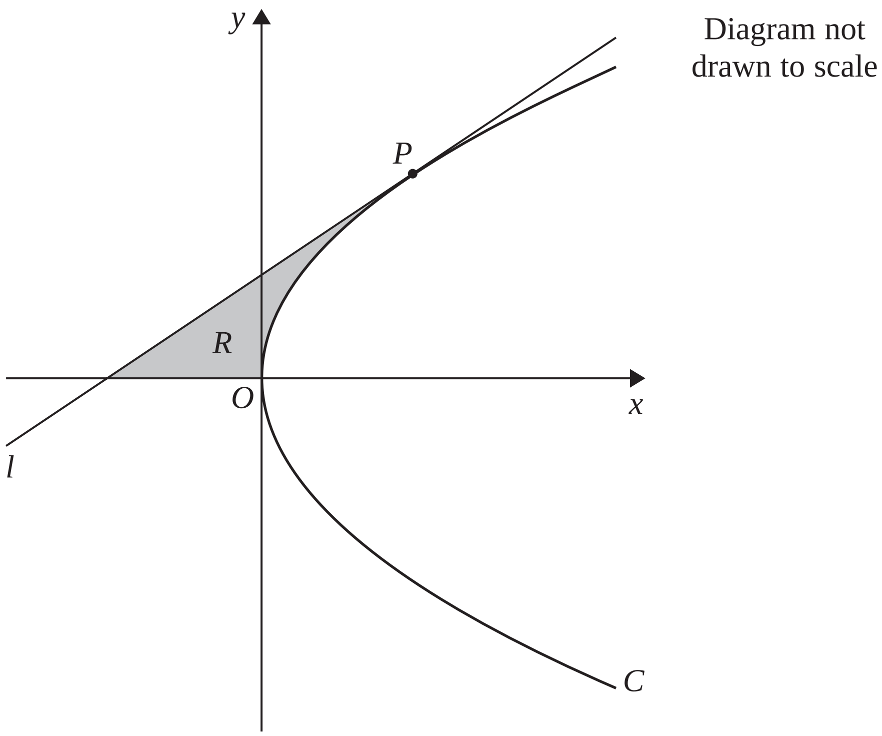


Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

Question 5 continued

Question 5 continued

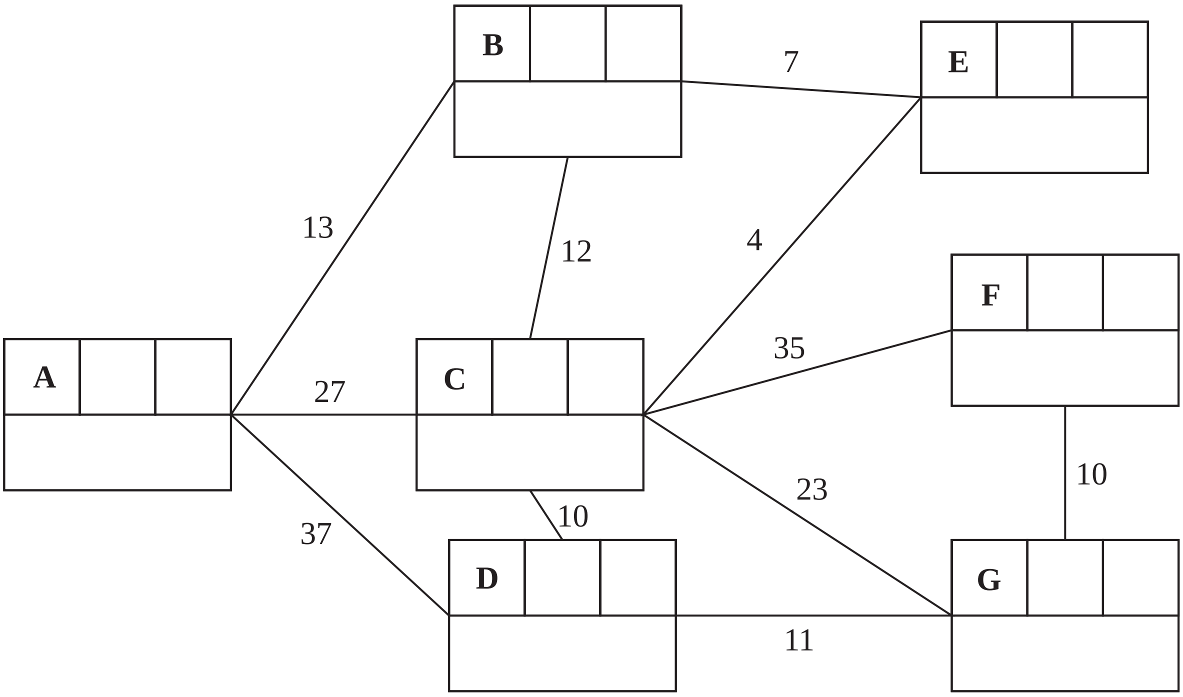
(Total for Question 5 is 12 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics question insert.

6.



Key:

Vertex	Order of labelling	Final value
Working values		

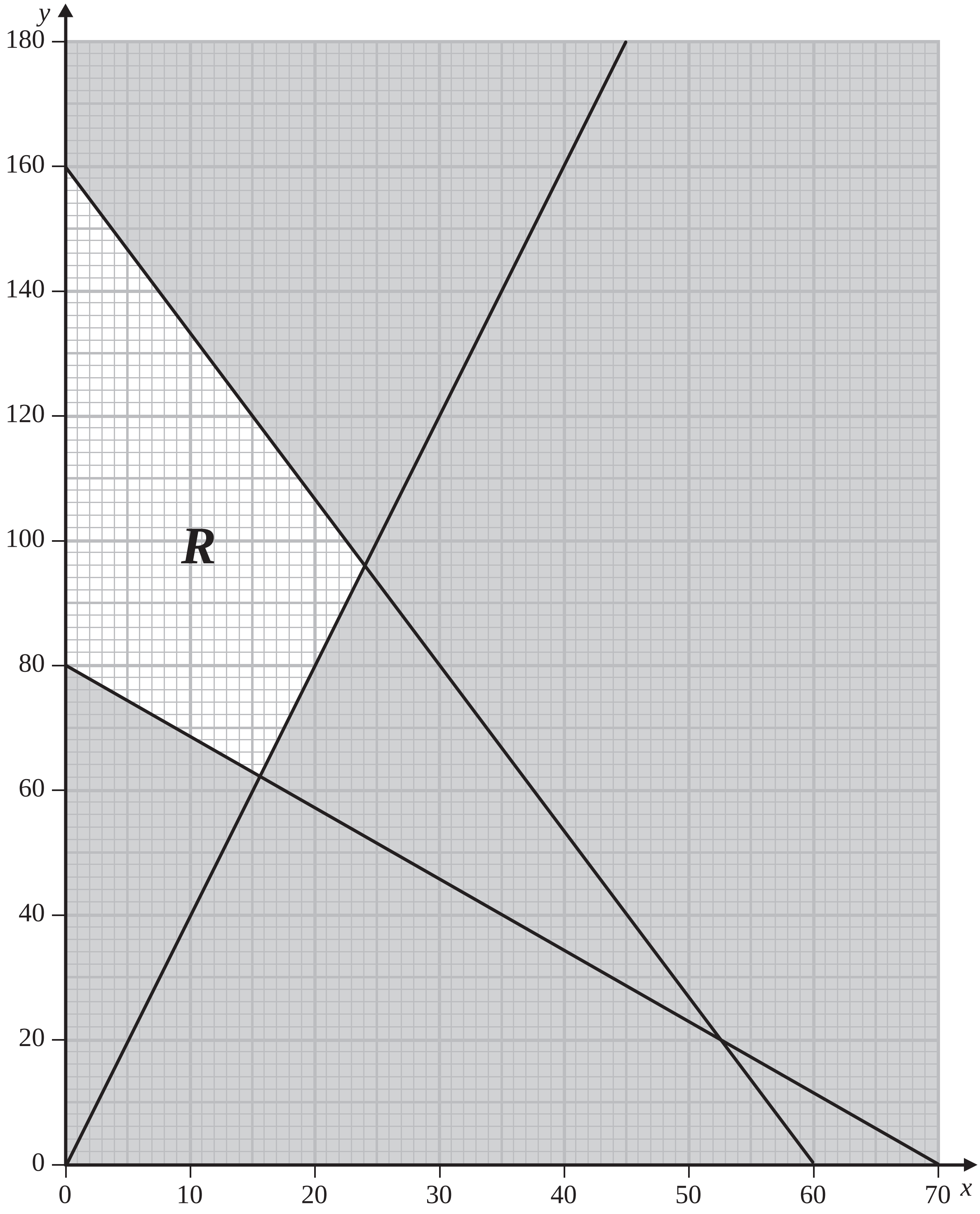
Shortest path: ______________________________

Length of shortest path: ______________________________

Question 6 continued

(Total for Question 6 is 12 marks)

7.



Question 7 continued

(Total for Question 7 is 7 marks)

8. (a) and (b)

(Total for Question 8 is 7 marks)

9.

(Total for Question 9 is 9 marks)

10.

(Total for Question 10 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2D: Section B Decision Mathematics 1

Sample Assessment Material for first teaching September 2017
Questions 6 – 10

Paper Reference
8FM0/2D

Do not return this document with the question paper.

Total Marks

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Pearson

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

6.

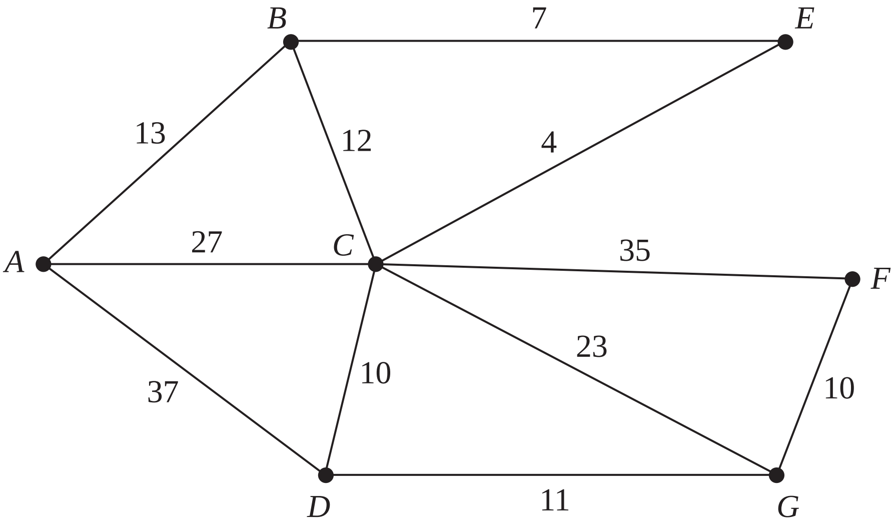


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra’s algorithm to find the shortest path from A to F. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel’s route.

(1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

(d) Calculate the length of BG. You must show your working.

(2)

(Total for Question 6 is 12 marks)

7.

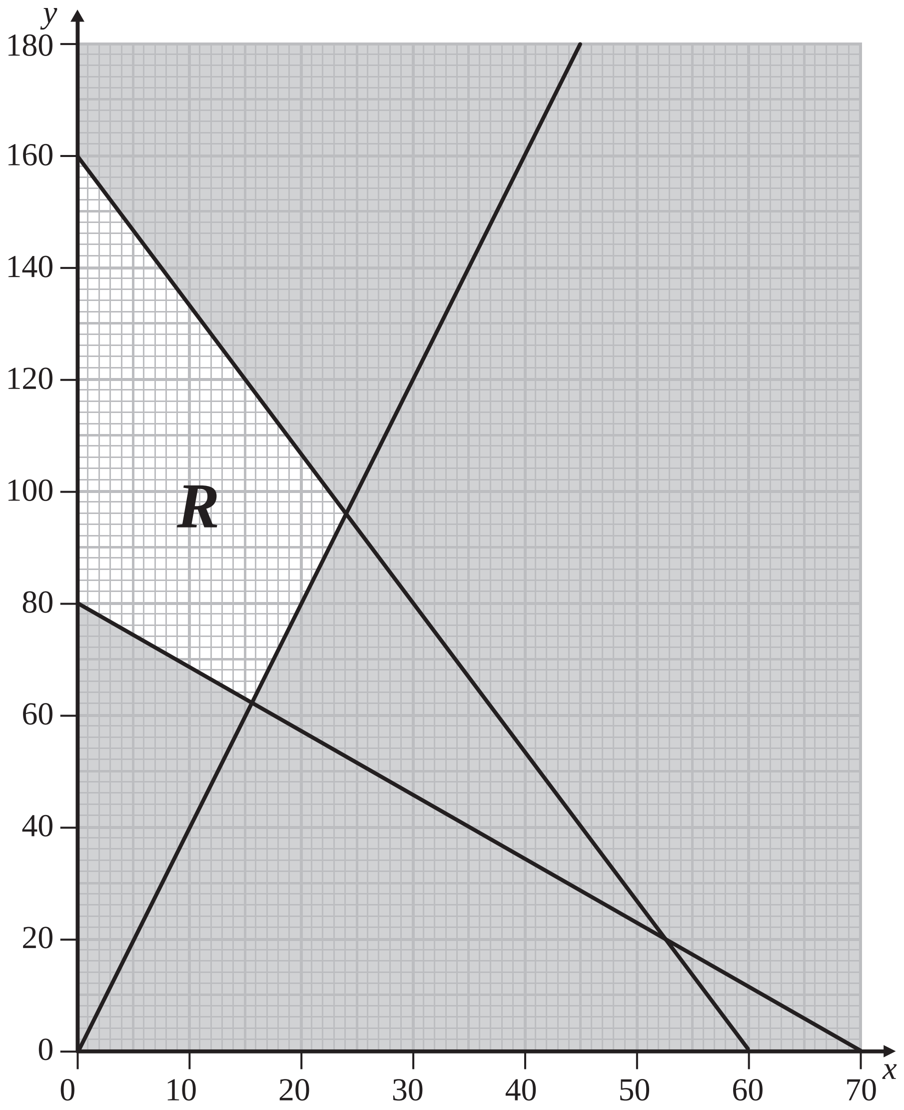


Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leqslant 480$$

$$8x + 7y \geqslant 560$$

$$y \geqslant 4x$$

$$x, y \geqslant 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 7 is 7 marks)

8.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	3	-
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) (i) Show that the project can be completed in 21 days, showing your working.

(ii) Identify the critical activities.

(4)

(Total for Question 8 is 7 marks)

9. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1$, 2^x, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 9 is 9 marks)

10. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 10 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS

Paper 2 Option D

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	$\sec x - \tan x = \dfrac{1}{\dfrac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes:

(a)

M1: Uses $\sec x = \dfrac{1}{\cos x}$ and the *t*-substitutions for both cos *x* and tan *x* to obtain an expression in terms of *t*

M1: Sorts out the sec *x* term, and puts over a common denominator of $1-t^2$

A1*: Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer

(b)

M1: Uses the *t*-substitution for sin *x* in both numerator and denominator

M1: Multiples through by $1+t^2$ in numerator and denominator

A1*: Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result

Question	Scheme	Marks	AOs
2	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step *h* required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2-1}{1^2+3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25\times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2-1.25}{1.25^2+1.25\times 3.5}\ \left(=\frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25\times\frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
(6 marks)			

Notes:

B1: Identifies the correct initial conditions and requirement for *h*

M1: Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0

M1: Applies the approximation formula with their values

A1ft: 3.5 or exact equivalent. Follow through their step value

M1: Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5

A1: Applies the approximation and interprets the result to give £396

Question	Scheme	Marks	AOs
3	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R}: x < -2\} \cup \{x \in \mathbb{R}: -1 < x < 0\} \cup \{x \in \mathbb{R}: x > 2\}$	M1 A1	2.2a 2.5
		(6)	
		(6 marks)	

Notes:

M1: Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side

M1: Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors

A1: At least 2 correct critical values found

A1: Exactly 4 correct critical values

M1: Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch, number line or any other means

A1: Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R}: x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$

Question	Scheme	Marks	AOs
4(a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\text{m}^2)$	A1	1.1b
		(4)	
	Alternative		
	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\text{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=\ldots$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\text{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
	4(b) Alternative		
	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB = \frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert = \frac{1}{2}\times 2\times\sqrt{10} = \sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+2\mathbf{k} - p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k}) \Rightarrow p=\frac{1}{2}$ and so height of tetrahedron is $h=\frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert = \frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V = 5 \times$ Volume of tetrahedron $= 5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$= \frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete Measurements in m may not be accurate The surface of the concrete tetrahedron may not be smooth Pockets of air may form when the concrete is being poured	B1	3.2b
		(1)	
		(10 marks)	

Question 4 notes:

Accept use of column vectors throughout

(a)

M1: Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*

M1: Any correct method for the triangle area is fine

M1: Finds $\mathbf{AB}$ and $\mathbf{AC}$ or any other appropriate pair of vectors to use in the vector product and attempts to use them

A1: Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent

(a) Alternative

M1: Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides

M1: May use different sides to those shown

M1: Correct full method to find the area of the triangle using their two sides

A1: $\frac{1}{2}\sqrt{35}$ or exact equivalent

Question 4 notes continued:

(b)

M1: Attempts volume of concrete by finding volume of tetrahedron with appropriate method

M1: Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted

A1: Correct value for the volume of concrete

M1: Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped

A1: $\frac{25}{3}$ only, ignore reference to units

(b) **Alternative**

M1: Notes (or works out using scalar products) that $-\mathbf{j}+\mathbf{3k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$

A1: Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$

M1: Solves $\mathbf{i}+\mathbf{j}+\mathbf{2k} - p(-\mathbf{j}+\mathbf{3k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k})$ to get the height of the tetrahedron

$$\left[(\mu=\lambda=)\ p=\frac{1}{2},\ \text{so } h=\frac{1}{2}\left|-\mathbf{j}+3\mathbf{k}\right|=\frac{1}{2}\sqrt{10}\right]$$

M1: Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference)

A1: $\frac{25}{3}$ only, ignore reference to units

(c)

B1: Any acceptable reason in context

Question	Scheme	Marks	AOs
5(a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 16$ so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{y}$	M1	2.2a
	$l:\ y - 8p = \left(\frac{8}{8p}\right)\left(x - 4p^2\right)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p - 3)(3p + 2) = 0 \Rightarrow p = \ldots$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \ldots$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}\mathrm{d}x$	M1	2.1
	$\int 4x^{\frac{1}{2}}\mathrm{d}x = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\ (+c)$ or $\frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
	5(c) Alternative 1		
	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12}\left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right)\mathrm{d}y$	M1	2.1
	$\int\left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right)\mathrm{d}y = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	
	5(c) Alternative 2		
	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts px-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9\left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}x$	M1	2.1
	$\int\left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right)\mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Question 5 notes:

(a)

B1: Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_p = 4p^2$ into $16x$ to obtain $64p^2$and concludes that *P* lies on *C*

(b)

M1: Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it

M1: Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient *m*, which is in terms of *p*.

Accept use of $8p = m(4p^2) + c$ with a clear attempt to find *c*

A1*: Obtains $py = x + 4p^2$ by **cso**

(c)

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$

A1: Finds that *l* cuts the *x*-axis at $x = -9$

M1: Fully correct method for finding the area of *R*

i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\text{d}x$

M1: Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$

A1: Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

(c) Alternative 1

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$

M1: Finds *l* as $x = \frac{3}{2}y - 9$

A1: Fully correct method for finding the area of *R*

M1: i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)\text{d}y$

M1: Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$,where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Question 5 notes continued:

(c) Alternative 2

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$

A1: Finds that *l* cuts the *x*-axis at $x = -9$

M1: Fully correct method for finding the area of *R*

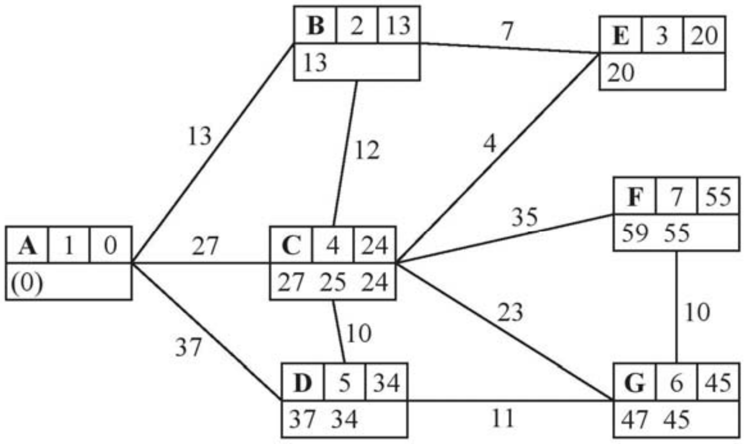
i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their} \left(\frac{2}{3}x + 6 \right) - \left(4x^{\frac{1}{2}} \right) \right) \mathrm{d}y$

M1: Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\left(\frac{2}{3}x + 6 \right) - \left(4x^{\frac{1}{2}} \right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Decision Mathematics 1 Mark Scheme (Section **B**)

Question	Scheme	Marks	AOs
6(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
		(1)	
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)

M1: For a larger number replaced by a smaller one in the working values boxes at C, D, F or G

A1: For all values correct (and in correct order) at A, B, C and D

A1: For all values correct (and in correct order) at E, F & G

A1: For the correct path

A1ft: For 55 or ft their final value at F

(b)

M1: For 3 correct pairings of the four odd nodes (A,B, D & G)

A1: At least two pairings and totals correct

A2: All three pairings and totals correct

A3ft: Selecting their shortest pairing, and stating that these arcs should be repeated

Question **6** notes continued:
(c) **B1ft:** For 213 or 189 + their shortest repeat **M1:** For translating the information in the question in to an equation involving x, $2x$ and 34 **A1:** For a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
7	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15 ± 1, 63 ± 2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and y = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
			(7 marks)

Notes:

M1: Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C

M1: Solving simultaneous equations

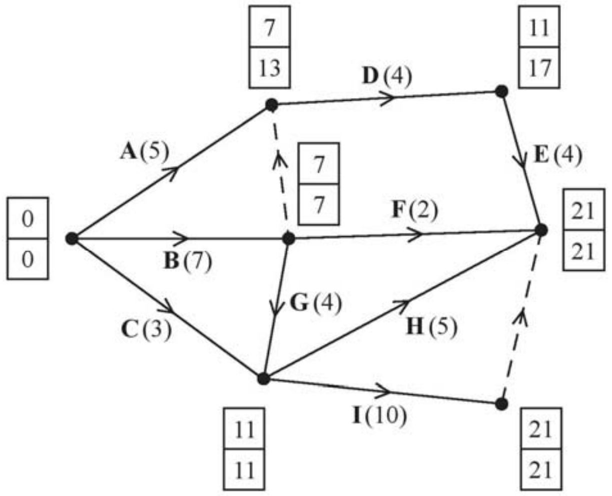
A1: cao

M1: Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$

M1: Testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C

A1: cao – deducing from tests which integer solution is both valid and optimal

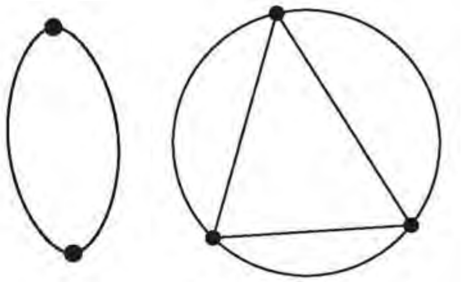
A1ft: Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
8(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
		M1 A1	2.1 1.1b
	The number(s) at the end of activity E indicate this project can be completed in 21 days	A1ft	2.2a
	Critical activities: B, G, I	A1	1.1b
		(4)	
			(7 marks)

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: All boxes completed, number generally increasing L to R (condone one “rogue”)
A1: All values cao
A1: Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes)
A1: Critical activities correct

Question	Scheme	Marks	AOs
9(a)	e.g. a graph cannot contain an odd number of odd nodes e.g. number of arcs $=\frac{1+3+4+4+5}{2}=8.5\notin\mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+\left(x+1\right)+\left(2^{x+1}-3\right)+\left(11-x\right)=2\left(18\right)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0\Rightarrow x=...$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0\Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1	2.4
		A1	2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
		(9 marks)	

Notes:

(a)
B1: Explanation referring to need for an even number of odd nodes oe

(b)
M1: Forming an equation involving the orders of the 5 odd nodes and 2(18)
M1: Simplifies to a quadratic in 2^x and attempts to solve
A1: 2 cao
M1: Construct an argument involving the order of the 5 nodes
A1: Explanation considering the number of odd nodes
A1: Deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: Interprets mathematical language to construct a disconnected graph
A1: Deduce a correct graph

Question	Scheme	Marks	AOs
10	Minimise $(C =)\ 25x+35y$	B1	3.3
	Subject to: $(500x+800y \geqslant 150\ 000 \Rightarrow)\ 5x+8y \geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y) \leqslant x \leqslant \frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b
			(5 marks)

Notes:

B1: A correct objective function + minimise
B1: Translate information in to a correct inequality
M1: For translating the information given into the LHS inequality
M1: For translating the information given in to the RHS inequality
A1: Simplifying to the correct inequalities

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2E: Further Statistics 1 and Further Mechanics 1

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2E

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.

(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6

(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.

(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.

(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.

(1)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The discrete random variable X has probability distribution given by

x	–1	0	1	2	3
$\mathrm{P}(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $\mathrm{E}(Y) = -4$ and $\mathrm{P}(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $\mathrm{E}(Y^2) = 75$

(b) find the exact value of $\mathrm{Var}(X)$

(2)

(c) Find $\mathrm{P}(Y > X)$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. Two car hire companies hire cars independently of each other.

Car Hire A hires cars at a rate of 2.6 cars per hour.

Car Hire B hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.
(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3
(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9
(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.
(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.
(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $\mathrm{P}(X = 1)$

(ii) $\mathrm{P}(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

5. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 5 continued

(Total for Question 5 is 10 marks)

6. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 6 continued

(Total for Question 6 is 6 marks)

7. [*In this question use* $g = 10\,\text{m s}^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\,\text{m s}^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \dfrac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\text{m s}^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\text{m s}^{-1}$.

(5)

Question 7 continued

(Total for Question 7 is 8 marks)

8. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \dfrac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \dfrac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 16 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Paper 2 Option E

Further Statistics 1 Mark Scheme (Section A)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1(a)</td><td>H_0: There is no association between language and gender</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>$\frac{54\times85}{150}=30.6$ *</td><td>B1*cso</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><td colspan="2" rowspan="2">Expected frequencies</td><td colspan="3">Language</td></tr>
<tr><td>French</td><td>Spanish</td><td>Mandarin</td></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>26.43…</td><td>23.4</td><td>15.16…</td></tr>
<tr><td>Female</td><td>34.56…</td><td>[30.6]</td><td>19.83…</td></tr>
</table>
</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(23-26.43)^2}{26.43}+...+\frac{(15-19.83)^2}{19.83}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>Awrt <u>3.6/3.7</u></td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01}=9.210$</td><td>M1</td><td>3.1b</td></tr>
<tr><td></td><td>As $\sum\frac{(O-E)^2}{E}<9.210$, the null hypothesis is not rejected</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(e)</td><td>Still not rejected since $\sum\frac{(O-E)^2}{E}<\chi^2_{2,0.1}=4.605$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4">(8 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">(a)
B1: For correct hypothesis in context</td></tr>
<tr><td colspan="4">(b)
B1*: For a correct calculation leading to the given answer and no errors seen</td></tr>
<tr><td colspan="4">(c)
M1: For attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1: For applying $\sum\frac{(O-E)^2}{E}$
A1: awrt 3.6 or 3.7</td></tr>
<tr><td colspan="4">(d)
M1: For using degrees of freedom to set up a χ^2 model critical value
A1: For correct comparison and conclusion</td></tr>
<tr><td colspan="4">(e)
A1ft: For correct conclusion with supporting reason</td></tr>
</table>

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5\text{E}(X)$	M1	3.1a
	$\text{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2 \quad \boxed{1}$		
	$\text{P}(Y \geqslant -3) = 0.45$ gives $\text{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\text{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45 \quad \boxed{2}$		
	$2a + b + 2c = 1 \quad \boxed{3}$	M1	1.1b
	$\begin{pmatrix}1&2&2\\2&0&1\\2&1&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1.2\\0.45\\1\end{pmatrix} \Rightarrow \begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}1&2&-2\\2&2&-3\\-2&-3&4\end{pmatrix}\begin{pmatrix}1.2\\0.45\\1\end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. $\boxed{3}-\boxed{2} \Rightarrow b + c = 0.55$ sub. $2(b + c)$ into $\boxed{1} \Rightarrow a = 0.1$ etc		
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	$[\text{Var}(Y) = 5^2\text{Var}(X)$ implies$]$ $\text{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\text{P}(Y > X) = \text{P}(2 - 5X > X) \rightarrow \text{P}(X < \frac{1}{3})$	M1	3.1a
	$\text{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
			(11 marks)

Notes:

(a)

M1: For using given information to find an expression for E(X) i.e. use of $\text{E}(Y) = 2 - 5\text{E}(X)$

M1: For use of $\sum x\text{P}(X = x) =$ '1.2'

M1: For use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \text{P}(X = x) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

Question 2 notes continued:

Another method for part (a) is:

M1: For using given information to find the probability distribution for Y leading to an expression for E(Y)

M1: For use of $\sum y\mathrm{P}(Y = y) = -4$

M1: For use of P($Y \geqslant -3$) = 0.45 to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \mathrm{P}(Y = y) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

(b)

M1: For use of Var(Y) = E(Y^2) – [E(Y)]^2 (may be implied by a correct answer)

A1: For use of Var(aX) = a^2 Var(X) to reach 2.36 or exact equivalent

(c)

M1: For rearranging to the form P($X < k$)

A1ft: 0.1’ + ‘025’ (provided their a and c and their $a + c$ are all probabilities)

Another method for part (c) is:

M1: For comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$

A1ft: ‘0.1’ + ‘025’ (provided their a and c and their $a + c$ are all probabilities)

Question	Scheme	Marks	AOs
3(a)	$X \sim \text{Po}(2.6)$ $\quad Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= \text{P}(X=2) \times \text{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **<u>0.0544</u>**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow \quad W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots.$ awrt **<u>0.205</u>**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **<u>0.648</u>**	A1	1.1b
		(2)	
(d)	**(i)** Mean = np = **<u>2.4</u>**	B1	1.1b
	(ii) Variance = $np(1 - p) = 2.3904$ awrt **<u>2.39</u>**	B1	1.1b
		(2)	
(e)	**(i)** [$D \sim \text{Po}(2.4)$ $\quad \text{P}(D \leqslant 4)$] $= 0.9041\ldots$ awrt **<u>0.904</u>**	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes:

(a)

M1: For $\text{P}(X=2) \times \text{P}(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)

A1: **awrt 0.0544**

(b)

M1: For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)

A1: **awrt 0.205**

(c)

M1: For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)

A1: **awrt 0.648**

(d)(i)

B1: For **2.4**

(d)(ii)

B1: For **awrt 2.39**

(e)(i)

B1: For **awrt 0.904**

(e)(ii)

B1: For a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
4(a)	**(i)** P(X = 1) = 0.34523… awrt **<u>0.345</u>**	B1	1.1b
	(ii) P($X \leqslant$ 4) = 0.98575… awrt **<u>0.986</u>**	B1	1.1b
		(2)	
(b)	$\frac{(0\times10)+1\times16+2\times7+3\times4+4\times2+(5\times0)+6\times1}{40}=1.4*$	B1*cso	1.1b
		(1)	
(c)	r = 40 × ‘0.34523…’ s = 40 × ‘1 – 0.986…’	M1	3.4
	r = **<u>13.81</u>** s = **<u>0.57</u>**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(10-9.86)^2}{9.86}+...+\frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **<u>1.1</u>**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since 1.10 < $\chi^2_{2,(0.05)}=5.991$). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		(6)	
		(11 marks)	

Notes:

(a)(i)
B1: awrt 0.345

(a)(ii)
B1: awrt 0.986

(b)
B1*: For a fully correct calculation leading to given answer with no errors seen

(c)
M1: For attempt at r or s (may be implied by correct answers)
A1ft: For both values correct (follow through their answers to part (a))

(d)
B1: For both hypotheses correct (lambda should not be defined so correct use of the model)
M1: For understanding the need to combine cells before calculating the test statistic (may be implied)
M1: For attempt to find the test statistic using $\chi^2=\sum\frac{(O-E)^2}{E}$
A1: awrt 1.1
B1: For realising that there are 2 degrees of freedom leading to a critical value of $\chi_2{}^2(0.05)=5.991$
A1: Concluding that a Poisson model is suitable for the number of mortgages approved each week

Further Mechanics 1 Mark Scheme (Section **B**)

Question	Scheme	Marks	AOs
5(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1*	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh \Rightarrow h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		(1)	
			(10 marks)

Notes:

(a)
M1: For a complete method to find v
A1: For a correct value (may be numerical)
M1: For a complete method to find u
A1: For a correct value (may be numerical)
M1: For finding both v and u and use of Newton's Law of Restitution
A1*: For the given answer

(b)
M1: For use of Newton's Law of Restitution to find rebound speed
M1: For a complete method to find h
A1: For 0.15 (m) oe

(c)
B1: For a clear description including reference to a quarter

Question	Scheme	Marks	AOs
6(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5\, g \times 20$	A1	1.1b
	= 58.25 = 58 (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20\, R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1ft	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
		(6 marks)	

Notes:

(a)
M1: For a difference in KE and PE
A1: For a correct expression
A1: For either 58 (2sf) or 58.3(3sf)

(b)
M1: For use of work-energy principle
A1ft: For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

(c)
B1: For variable resistance oe

Question	Scheme	Marks	AOs
7(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force × Speed = 30 × 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force × Speed = 70 × 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)
B1: For force = 30 seen
M1: For use of $P = Fv$
A1ft: For 120 (W), follow through on their '30'

(b)
M1: For resolving parallel to the slope with correct no. of terms and 60g resolved
A1: For a correct equation
A1: For $F = 70$
M1: For use of $P = Fv$
A1ft: For 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
8(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	

(16 marks)

Notes:

(a)
M1: For use of CLM, with correct no. of terms, condone sign errors
A1: For a correct equation
M1: For use of Newton's Law of Restitution, with e on the correct side
A1: For a correct equation
M1: For setting up *two* equations and solving their equations for v
A1: For a correct expression for v

(b)
M1: For use of an appropriate inequality
A1: For a complete range of values of e

(c)
M1: For solving their equations for w
A1: For the given answer

Question **8** notes continued:
(d) **M1:** For substituting $e = \frac{5}{9}$ into their v and w **A1:** For correct expressions for v and w **M1:** For use of Newton's Law of Restitution, with e on the correct side **M1:** For use of appropriate inequality **A1:** For a correct inequality **A1:** For a correct range

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2F: Further Statistics 1 and Decision Mathematics 1

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2F

You must have:
Decision Mathematics 1 question insert for Section B
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.

(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6

(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.

(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.

(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.

(1)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$\mathrm{P}(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $\mathrm{E}(Y) = -4$ and $\mathrm{P}(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $\mathrm{E}(Y^2) = 75$

(b) find the exact value of $\mathrm{Var}(X)$

(2)

(c) Find $\mathrm{P}(Y > X)$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

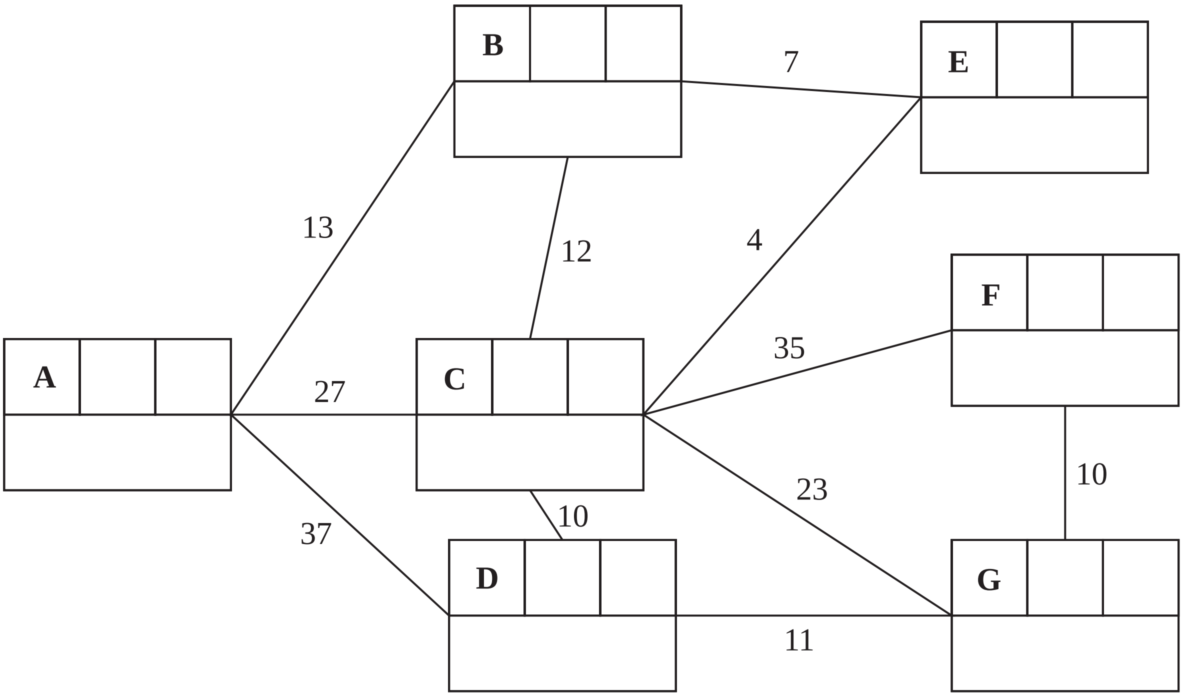
(Total for Question 4 is 11 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics 1 question insert.

5.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path: ______________________________

Length of shortest path: ______________________________

DO NOT WRITE IN THIS AREA

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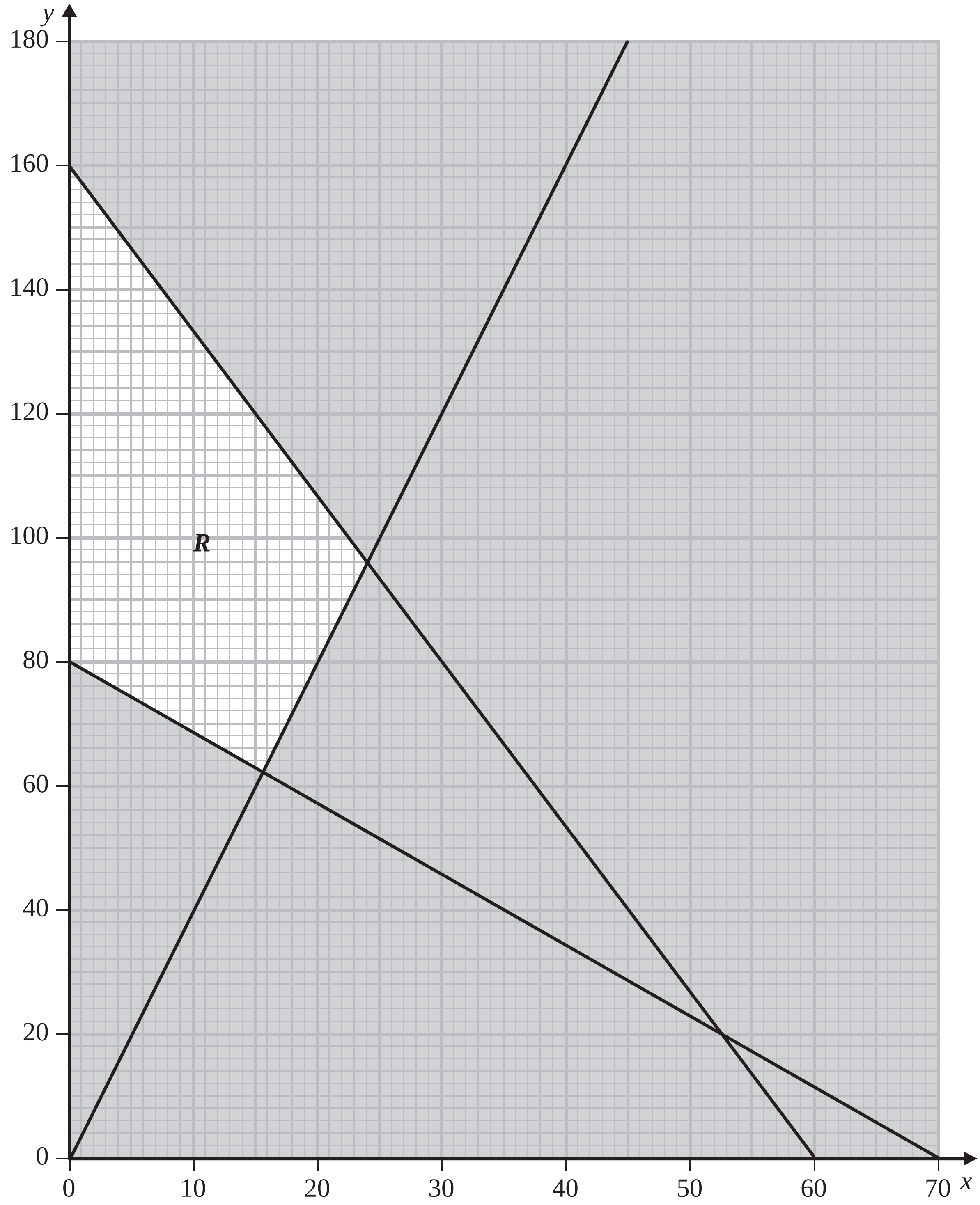
Question 5 continued

(Total for Question 5 is 12 marks)

6.



Question 6 continued

(Total for Question 6 is 7 marks)

7. (a) and (b)

(Total for Question 7 is 7 marks)

8.

(Total for Question 8 is 9 marks)

9.

(Total for Question 9 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2F: Section B Decision Mathematics 1

Sample Assessment Material for first teaching September 2017

Paper Reference
8FM0/2F

Decision Mathematics 1 question insert for Section B
Do not return this document with the question paper.

Turn over ▶

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Pearson

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

5.

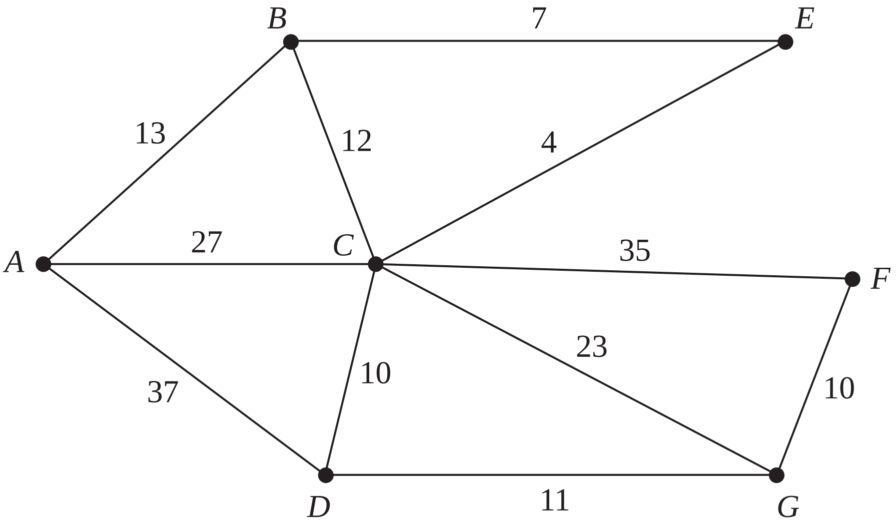


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

(d) Calculate the length of BG. You must show your working.

(2)

(Total for Question 5 is 12 marks)

6.

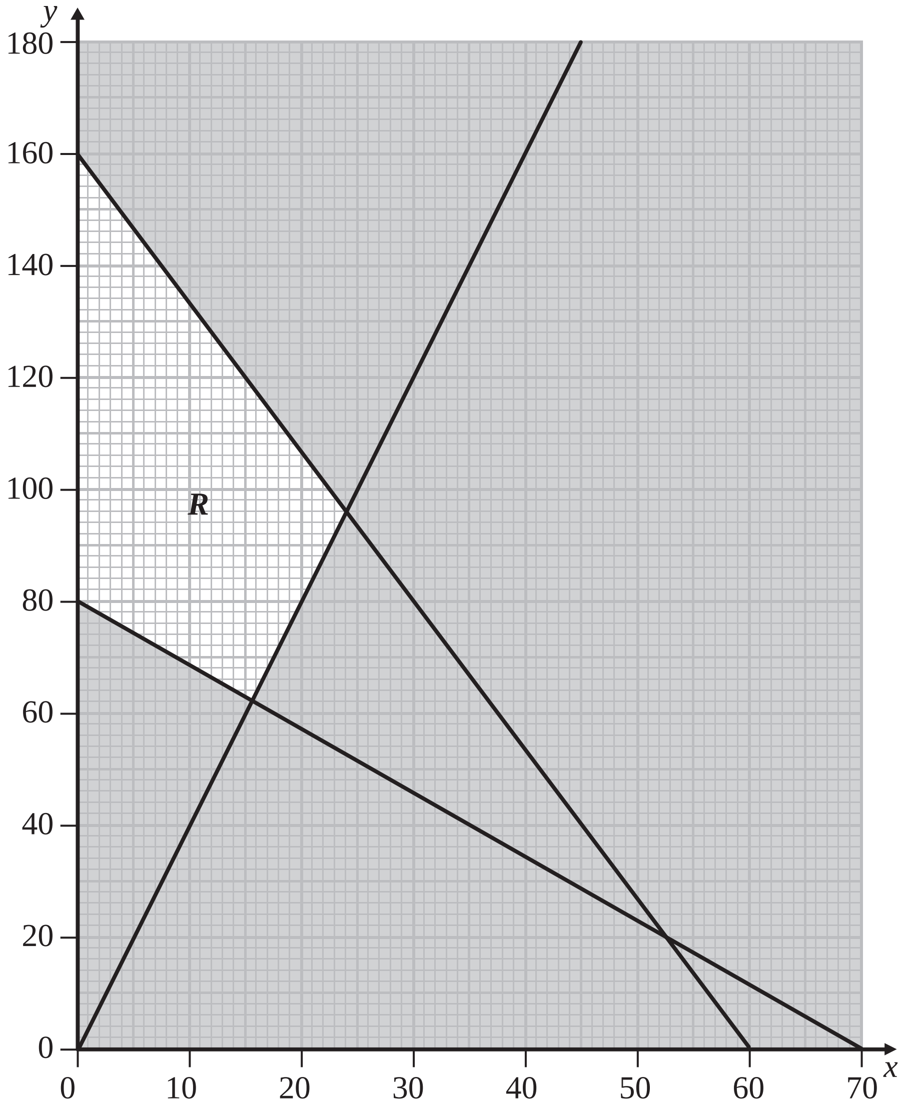


Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leqslant 480$$

$$8x + 7y \geqslant 560$$

$$y \geqslant 4x$$

$$x, y \geqslant 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 6 is 7 marks)

7.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	3	-
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) (i) Show that the project can be completed in 21 days, showing your working.

(ii) Identify the critical activities.

(4)

(Total for Question 7 is 7 marks)

8. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1$, 2^x, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 8 is 9 marks)

9. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 9 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS

Paper 2 Option **F**

Further Statistics 1 Mark Scheme (Section A)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="2">**1(a)**</td><td>H_0: There is no association between language and gender</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>**(1)**</td><td></td></tr>
<tr><td rowspan="2">**(b)**</td><td>$\frac{54\times85}{150}=30.6$ *</td><td>B1*cso</td><td>1.1b</td></tr>
<tr><td></td><td>**(1)**</td><td></td></tr>
<tr><td rowspan="4">**(c)**</td><td>
<table>
<tr><td colspan="2" rowspan="2">Expected frequencies</td><td colspan="3">Language</td></tr>
<tr><td>French</td><td>Spanish</td><td>Mandarin</td></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>26.43…</td><td>23.4</td><td>15.16…</td></tr>
<tr><td>Female</td><td>34.56…</td><td>[30.6]</td><td>19.83…</td></tr>
</table>
</td><td>M1</td><td>2.1</td></tr>
<tr><td>$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(23-26.43)^2}{26.43}+...+\frac{(15-19.83)^2}{19.83}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Awrt **3.6/3.7**</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>**(3)**</td><td></td></tr>
<tr><td rowspan="3">**(d)**</td><td>Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01}=9.210$</td><td>M1</td><td>3.1b</td></tr>
<tr><td>As $\sum\frac{(O-E)^2}{E}<9.210$, the null hypothesis is not rejected</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td>**(2)**</td><td></td></tr>
<tr><td rowspan="2">**(e)**</td><td>Still not rejected since $\sum\frac{(O-E)^2}{E}<\chi^2_{2,0.1}=4.605$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>**(1)**</td><td></td></tr>
<tr><td colspan="4">**(8 marks)**</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">**(a)**
B1: For correct hypothesis in context</td></tr>
<tr><td colspan="4">**(b)**
B1*: For a correct calculation leading to the given answer and no errors seen</td></tr>
<tr><td colspan="4">**(c)**
M1: For attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1: For applying $\sum\frac{(O-E)^2}{E}$
A1: awrt 3.6 or 3.7</td></tr>
<tr><td colspan="4">**(d)**
M1: For using degrees of freedom to set up a χ^2 model critical value
A1: For correct comparison and conclusion</td></tr>
<tr><td colspan="4">**(e)**
A1ft: For correct conclusion with supporting reason</td></tr>
</table>

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5\mathrm{E}(X)$	M1	3.1a
	$\mathrm{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2 \quad \boxed{1}$		
	$\mathrm{P}(Y \geqslant -3) = 0.45$ gives $\mathrm{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\mathrm{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45 \quad \boxed{2}$		
	$2a + b + 2c = 1 \quad \boxed{3}$	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix}\begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. $\boxed{3} - \boxed{2} \Rightarrow b + c = 0.55$ sub. $2(b+c)$ into $\boxed{1} \Rightarrow a = 0.1$ etc		
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\mathrm{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	$[\mathrm{Var}(Y) = 5^2\mathrm{Var}(X)$ implies$]$ $\mathrm{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\mathrm{P}(Y > X) = \mathrm{P}(2 - 5X > X) \rightarrow \mathrm{P}(X < \frac{1}{3})$	M1	3.1a
	$\mathrm{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
			(11 marks)

Notes:

(a)

M1: For using given information to find an expression for E(X) i.e. use of $\mathrm{E}(Y) = 2 - 5\mathrm{E}(X)$

M1: For use of $\sum x\mathrm{P}(X = x) =$ ‘1.2’

M1: For use of $\mathrm{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \mathrm{P}(X = x) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

Question 2 notes continued:

Another method for part (a) is:

M1: For using given information to find the probability distribution for Y leading to an expression for E(Y)

M1: For use of $\sum y\text{P}(Y = y) = -4$

M1: For use of P($Y \geqslant -3$) = 0.45 to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \text{P}(Y = y) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

(b)

M1: For use of Var(Y) = E(Y^2) – [E(Y)]^2 (may be implied by a correct answer)

A1: For use of Var(aX) = a^2 Var(X) to reach 2.36 or exact equivalent

(c)

M1: For rearranging to the form P($X < k$)

A1ft: 0.1’ + ‘025’ (provided their a and c and their $a + c$ are all probabilities)

Another method for part (c) is:

M1: For comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$

A1ft: ‘0.1’ + ‘025’ (provided their a and c and their $a + c$ are all probabilities)

Question	Scheme	Marks	AOs
3(a)	$X \sim \text{Po}(2.6)$ $Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= \text{P}(X{=}2) \times \text{P}(Y{=}2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **<u>0.0544</u>**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots.$ awrt **<u>0.205</u>**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6{+}1.2){\times}2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **<u>0.648</u>**	A1	1.1b
		(2)	
(d)	**(i)** Mean = np = **<u>2.4</u>**	B1	1.1b
	(ii) Variance = $np(1 - p) = 2.3904$ awrt **<u>2.39</u>**	B1	1.1b
		(2)	
(e)	**(i)** [$D \sim \text{Po}(2.4)$ $\text{P}(D \leqslant 4)$] $= 0.9041\ldots$ awrt **<u>0.904</u>**	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes:

(a)

M1: For $\text{P}(X{=}2) \times \text{P}(Y{=}2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)

A1: **awrt 0.0544**

(b)

M1: For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)

A1: **awrt 0.205**

(c)

M1: For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)

A1: **awrt 0.648**

(d)(i)

B1: For **2.4**

(d)(ii)

B1: For **awrt 2.39**

(e)(i)

B1: For **awrt 0.904**

(e)(ii)

B1: For a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
4(a)	**(i)** P(X = 1) = 0.34523… awrt **<u>0.345</u>**	B1	1.1b
	(ii) P($X \leqslant$ 4) = 0.98575… awrt **<u>0.986</u>**	B1	1.1b
		(2)	
(b)	$\frac{(0\times10)+1\times16+2\times7+3\times4+4\times2+(5\times0)+6\times1}{40}=1.4^{*}$	B1*cso	1.1b
		(1)	
(c)	r = 40 × '0.34523…' s = 40 × '1 – 0.986…'	M1	3.4
	r = **<u>13.81</u>** s = **<u>0.57</u>**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(10-9.86)^2}{9.86}+...+\frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **<u>1.1</u>**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since 1.10 < $\chi^2_{2,(0.05)}=5.991$). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		(6)	
			(11 marks)

Notes:

(a)(i)
B1: awrt 0.345

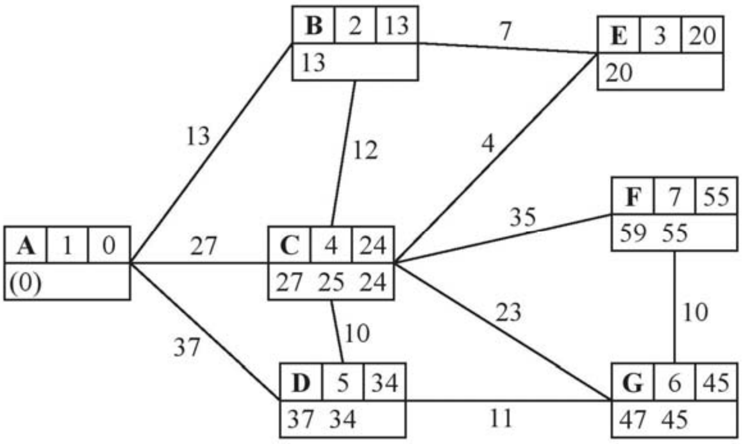
(a)(ii)
B1: awrt 0.986

(b)
B1*: For a fully correct calculation leading to given answer with no errors seen

(c)
M1: For attempt at r or s (may be implied by correct answers)
A1ft: For both values correct (follow through their answers to part (a))

(d)
B1: For both hypotheses correct (lambda should not be defined so correct use of the model)
M1: For understanding the need to combine cells before calculating the test statistic (may be implied)
M1: For attempt to find the test statistic using $\chi^2=\sum\frac{(O-E)^2}{E}$
A1: awrt 1.1
B1: For realising that there are 2 degrees of freedom leading to a critical value of $\chi_2{}^2(0.05)=5.991$
A1: Concluding that a Poisson model is suitable for the number of mortgages approved each week

Decision Mathematics 1 Mark Scheme (Section **B**)

Question	Scheme	Marks	AOs
5(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
		(1)	
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)

M1: For a larger number replaced by a smaller one in the working values boxes at C, D, F or G

A1: For all values correct (and in correct order) at A, B, C and D

A1: For all values correct (and in correct order) at E, F & G

A1: For the correct path

A1ft: For 55 or ft their final value at F

(b)

M1: For 3 correct pairings of the four odd nodes (A,B, D & G)

A1: At least two pairings and totals correct

A2: All three pairings and totals correct

A3ft: Selecting their shortest pairing, and stating that these arcs should be repeated

Question **5** notes continued:
(c) **B1ft:** For 213 or 189 + their shortest repeat **M1:** For translating the information in the question in to an equation involving x, $2x$ and 34 **A1:** For a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
6	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15 ± 1, 63 ± 2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and y = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
			(7 marks)

Notes:

M1: Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C

M1: Solving simultaneous equations

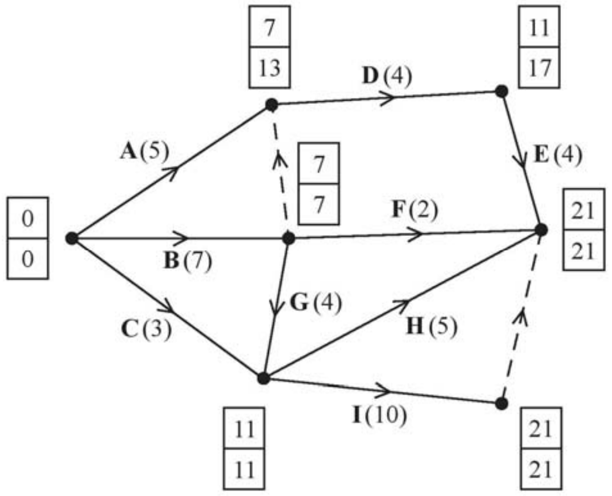
A1: cao

M1: Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$

M1: Testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C

A1: cao – deducing from tests which integer solution is both valid and optimal

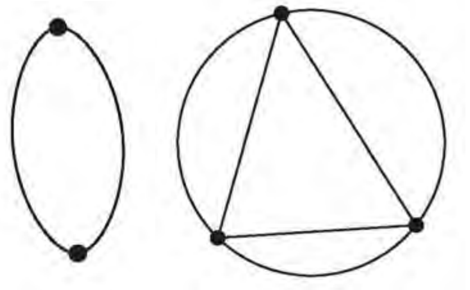
A1ft: Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
7(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
		M1 A1	2.1 1.1b
	The number(s) at the end of activity E indicate this project can be completed in 21 days	A1ft	2.2a
	Critical activities: B, G, I	A1	1.1b
		(4)	
			(7 marks)

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: All boxes completed, number generally increasing L to R (condone one "rogue")
A1: All values cao
A1: Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes)
A1: Critical activities correct

Question	Scheme	Marks	AOs
8(a)	e.g. a graph cannot contain an odd number of odd nodes e.g. number of arcs $=\frac{1+3+4+4+5}{2}=8.5\notin\mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)\ +\left(x+1\right)\ +\left(2^{x+1}-3\right)+\left(11-x\right)=2\left(18\right)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0\Rightarrow x=...$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0\Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1	2.4
		A1	2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
		(9 marks)	

Notes:

(a)
B1: Explanation referring to need for an even number of odd nodes oe

(b)
M1: Forming an equation involving the orders of the 5 odd nodes and 2(18)
M1: Simplifies to a quadratic in 2^x and attempts to solve
A1: 2 cao
M1: Construct an argument involving the order of the 5 nodes
A1: Explanation considering the number of odd nodes
A1: Deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: Interprets mathematical language to construct a disconnected graph
A1: Deduce a correct graph

Question	Scheme	Marks	AOs
9	Minimise $(C =)\ 25x + 35y$	B1	3.3
	Subject to: $(500x + 800y \geqslant 150\ 000 \Rightarrow)\ 5x + 8y \geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y) \leqslant x \leqslant \frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b
			(5 marks)

Notes:

B1: A correct objective function + minimise
B1: Translate information in to a correct inequality
M1: For translating the information given into the LHS inequality
M1: For translating the information given in to the RHS inequality
A1: Simplifying to the correct inequalities

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2G: Further Statistics 1 and Further Statistics 2

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2G

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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S 5 8 5 3 3 A 0 1 3 0

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

1. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language		
		French	**Spanish**	**Mandarin**
Gender	**Male**	23	22	20
	Female	38	32	15

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.
(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6
(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.
(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.
(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.
(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$\mathrm{P}(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $\mathrm{E}(Y) = -4$ and $\mathrm{P}(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $\mathrm{E}(Y^2) = 75$

(b) find the exact value of $\mathrm{Var}(X)$

(2)

(c) Find $\mathrm{P}(Y > X)$

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 11 marks)

3. Two car hire companies hire cars independently of each other.

Car Hire A hires cars at a rate of 2.6 cars per hour.

Car Hire B hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

5. In a gymnastics competition, two judges scored each of 8 competitors on the vault.

Competitor	A	B	C	D	E	F	G	H
Judge 1's scores	4.6	9.1	8.4	8.8	9.0	9.5	9.2	9.4
Judge 2's scores	7.8	8.8	8.6	8.5	9.1	9.6	9.0	9.3

(a) Calculate Spearman's rank correlation coefficient for these data.
(4)

(b) Stating your hypotheses clearly, test at the 1% level of significance, whether or not the two judges are generally in agreement.
(4)

(c) Give a reason to support the use of Spearman's rank correlation coefficient in this case.
(1)

The judges also scored the competitors on the beam.

Spearman's rank correlation coefficient for their ranks on the beam was found to be 0.952

(d) Compare the judges' ranks on the vault with their ranks on the beam.
(1)

Question 5 continued

(Total for Question 5 is 10 marks)

6. The continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} \frac{1}{18}(11 - 2x) & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find P(X < 3)

(2)

(b) State, giving a reason, whether the upper quartile of X is greater than 3, less than 3 or equal to 3

(1)

Given that $\mathrm{E}(X) = \frac{9}{4}$

(c) use algebraic integration to find Var(X)

(3)

The cumulative distribution function of X is given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{18}(11x - x^2 + c) & 1 \leqslant x \leqslant 4 \\ 1 & x > 4 \end{cases}$$

(d) Show that $c = -10$

(2)

(e) Find the median of X, giving your answer to 3 significant figures.

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7. A scientist wants to develop a model to describe the relationship between the average daily temperature, x °C, and a household's daily energy consumption, y kWh, in winter.

A random sample of the average temperature and energy consumption are taken from 10 winter days and are summarised below.

$$\sum x = 12 \qquad \sum x^2 = 24.76 \qquad \sum y = 251 \qquad \sum y^2 = 6341 \qquad \sum xy = 284.8$$

$$S_{xx} = 10.36 \qquad S_{yy} = 40.9$$

(a) Find the product moment correlation coefficient between y and x.

(2)

(b) Find the equation of the regression line of y on x in the form $y = a + bx$

(3)

(c) Use your equation to estimate the daily energy consumption when the average daily temperature is 2 °C

(1)

(d) Calculate the residual sum of squares (RSS).

(2)

The table shows the residual for each value of x.

x	–0.4	–0.2	0.3	0.8	1.1	1.4	1.8	2.1	2.5	2.6
Residual	–0.63	–0.32	–0.52	–0.73	0.74	2.22	1.84	0.32	f	–1.88

(e) Find the value of f.

(2)

(f) By considering the signs of the residuals, explain whether or not the linear regression model is a suitable model for these data.

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. The continuous random variable X is uniformly distributed over the interval [–3, 5].

(a) Sketch the probability density function f(x) of X.

(2)

(b) Find the value of k such that $\text{P}(X < 2[k - X]) = 0.25$

(3)

(c) Use algebraic integration to show that $\text{E}(X^3) = 17$

(3)

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Paper 2 Option G

Further Statistics 1 Mark Scheme (Section A)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="2">1(a)</td><td>H_0: There is no association between language and gender</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="2">(b)</td><td>$\frac{54 \times 85}{150} = 30.6 \quad *$</td><td>B1*cso</td><td>1.1b</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td rowspan="4">(c)</td><td>
<table>
<tr><th colspan="2" rowspan="2">Expected frequencies</th><th colspan="3">Language</th></tr>
<tr><th>French</th><th>Spanish</th><th>Mandarin</th></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>26.43…</td><td>23.4</td><td>15.16…</td></tr>
<tr><td>Female</td><td>34.56…</td><td>[30.6]</td><td>19.83…</td></tr>
</table>
</td><td>M1</td><td>2.1</td></tr>
<tr><td>$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + ... + \frac{(15-19.83)^2}{19.83}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Awrt <u>3.6/3.7</u></td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td rowspan="3">(d)</td><td>Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$</td><td>M1</td><td>3.1b</td></tr>
<tr><td>As $\sum \frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="2">(e)</td><td>Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">(a)
B1: For correct hypothesis in context</td></tr>
<tr><td colspan="4">(b)
B1*: For a correct calculation leading to the given answer and no errors seen</td></tr>
<tr><td colspan="4">(c)
M1: For attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies
M1: For applying $\sum \frac{(O-E)^2}{E}$
A1: awrt 3.6 or 3.7</td></tr>
<tr><td colspan="4">(d)
M1: For using degrees of freedom to set up a χ^2 model critical value
A1: For correct comparison and conclusion</td></tr>
<tr><td colspan="4">(e)
A1ft: For correct conclusion with supporting reason</td></tr>
</table>

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5\mathrm{E}(X)$	M1	3.1a
	$\mathrm{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2 \qquad \boxed{1}$		
	$\mathrm{P}(Y \geqslant -3) = 0.45$ gives $\mathrm{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\mathrm{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45 \qquad \boxed{2}$		
	$2a + b + 2c = 1 \qquad \boxed{3}$	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix}\begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. $\boxed{3} - \boxed{2} \Rightarrow b + c = 0.55$ sub. $2(b + c)$ into $\boxed{1} \Rightarrow a = 0.1$ etc		
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\mathrm{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	$[\mathrm{Var}(Y) = 5^2\mathrm{Var}(X)$ implies$]$ $\mathrm{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\mathrm{P}(Y > X) = \mathrm{P}(2 - 5X > X) \rightarrow \mathrm{P}(X < \tfrac{1}{3})$	M1	3.1a
	$\mathrm{P}(X < \tfrac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
		(11 marks)	

Notes:

(a)

M1: For using given information to find an expression for $\mathrm{E}(X)$ i.e. use of $\mathrm{E}(Y) = 2 - 5\mathrm{E}(X)$

M1: For use of $\sum x\mathrm{P}(X = x) =$ '1.2'

M1: For use of $\mathrm{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \mathrm{P}(X = x) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

Question 2 notes continued:
Another method for part (a) is: **M1:** For using given information to find the probability distribution for Y leading to an expression for E(Y) **M1:** For use of $\sum y\mathrm{P}(Y = y) = -4$ **M1:** For use of P($Y \geqslant -3$) = 0.45 to set up the argument for solving by forming an equation in a and c **M1:** For use of $\sum \mathrm{P}(Y = y) = 1$ **M1:** For solving their 3 linear equations (matrix or elimination) **A1:** For any 2 of a, b or c correct **A1:** For all 3 correct values
(b) **M1:** For use of Var(Y) = E(Y^2) – [E(Y)]^2 (may be implied by a correct answer) **A1:** For use of Var(aX) = a^2 Var(X) to reach 2.36 or exact equivalent
(c) **M1:** For rearranging to the form P($X < k$) **A1ft:** 0.1' + '025' (provided their a and c and their $a + c$ are all probabilities) **Another method for part (c) is:** **M1:** For comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$ **A1ft:** '0.1' + '025' (provided their a and c and their $a + c$ are all probabilities)

Question	Scheme	Marks	AOs
3(a)	$X \sim \text{Po}(2.6)$ $Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) = $\text{P}(X=2) \times \text{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	= 0.05444… awrt **<u>0.0544</u>**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow \ W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots.$ awrt **<u>0.205</u>**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **<u>0.648</u>**	A1	1.1b
		(2)	
(d)	**(i)** Mean = np = **<u>2.4</u>**	B1	1.1b
	(ii) Variance = $np(1 - p)$ = 2.3904 awrt **<u>2.39</u>**	B1	1.1b
		(2)	
(e)	**(i)** [$D \sim \text{Po}(2.4)$ $\text{P}(D \leqslant 4)$] = 0.9041… awrt **<u>0.904</u>**	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
		(10 marks)	

Notes:

(a)

M1: For $\text{P}(X=2) \times \text{P}(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)

A1: **awrt 0.0544**

(b)

M1: For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)

A1: **awrt 0.205**

(c)

M1: For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)

A1: **awrt 0.648**

(d)(i)

B1: For **2.4**

(d)(ii)

B1: For **awrt 2.39**

(e)(i)

B1: For **awrt 0.904**

(e)(ii)

B1: For a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
4(a)	**(i)** $P(X = 1) = 0.34523\ldots$ awrt **0.345**	B1	1.1b
	(ii) $P(X \leqslant 4) = 0.98575\ldots$ awrt **0.986**	B1	1.1b
		(2)	
(b)	$\frac{(0\times10)+1\times16+2\times7+3\times4+4\times2+(5\times0)+6\times1}{40}=1.4*$	B1*cso	1.1b
		(1)	
(c)	$r = 40 \times$ '0.34523…' $s = 40 \times$ '1 – 0.986…'	M1	3.4
	$r =$ **13.81** $s =$ **0.57**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2 = \sum\frac{(O-E)^2}{E} = \frac{(10-9.86)^2}{9.86} + \ldots + \frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **1.1**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since $1.10 < \chi^2_{2,(0.05)} = 5.991$). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		(6)	
			(11 marks)

Notes:

(a)(i)
B1: awrt 0.345

(a)(ii)
B1: awrt 0.986

(b)
B1*: For a fully correct calculation leading to given answer with no errors seen

(c)
M1: For attempt at r or s (may be implied by correct answers)
A1ft: For both values correct (follow through their answers to part (a))

(d)
B1: For both hypotheses correct (lambda should not be defined so correct use of the model)
M1: For understanding the need to combine cells before calculating the test statistic (may be implied)
M1: For attempt to find the test statistic using $\chi^2 = \sum\frac{(O-E)^2}{E}$
A1: awrt 1.1
B1: For realising that there are 2 degrees of freedom leading to a critical value of $\chi_2^2(0.05) = 5.991$
A1: Concluding that a Poisson model is suitable for the number of mortgages approved each week

Further Statistics 2 Mark Scheme (Section B)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td>
<table>
<tr><th>Competitor</th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th><th>G</th><th>H</th></tr>
<tr><td>**Judge 1’s ranks**</td><td>8</td><td>4</td><td>7</td><td>6</td><td>5</td><td>1</td><td>3</td><td>2</td></tr>
<tr><td>**Judge 2’s ranks**</td><td>8</td><td>5</td><td>6</td><td>7</td><td>3</td><td>1</td><td>4</td><td>2</td></tr>
<tr><td>d^2</td><td>0</td><td>1</td><td>1</td><td>1</td><td>4</td><td>0</td><td>1</td><td>0</td></tr>
</table>
</td><td>M1
M1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>$\sum d^2 = 8$
$r_s = 1 - \frac{6 \times 8}{8(64 - 1)}$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td></td><td>$r_s = 0.90476\ldots$ awrt **0.905**</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>**(4)**</td><td></td></tr>
<tr><td>(b)</td><td>H_0: $\rho_s = 0$ H_1: $\rho_s > 0$</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>Critical value $\rho_s = 0.8333$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$r_s = 0.905$ lies in the critical region/reject H_0</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>The two judges are in agreement.</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td></td><td>**(4)**</td><td></td></tr>
<tr><td>(c)</td><td>E.g. The data is unlikely to be from a bivariate normal distribution (competitor A)/The emphasis here is on the ranks and not the individual scores.</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>**(1)**</td><td></td></tr>
<tr><td>(d)</td><td>Both show positive correlation, but the judges agree more on the beam (since 0.952 is closer to 1)</td><td>B1</td><td>2.2b</td></tr>
<tr><td></td><td></td><td>**(1)**</td><td></td></tr>
<tr><td colspan="4">**(10 marks)**</td></tr>
</table>

Notes:

(a)

M1: For an attempt to rank at least one row (at least four correct)

M1: For an attempt at d^2 row for their ranks

M1: Dependent on 1st M1 for use of $r_s = 1 - \frac{6 \times 8}{8(64 - 1)}$ with their $\sum d^2$

A1: For awrt 0.905

(b)

B1: Both hypotheses stated in terms of ρ_s

B1: For correct critical value

M1: For comparing their ‘0.905’ with their ‘0.8333’

A1: For a correct contextual conclusion with no contradictions seen

(c)

B1: For a correct explanation to support the use of Spearman

(d)

B1: For a correct comparison of the correlation coefficients

Question	Scheme	Marks	AOs
6(a)	P(X < 3) = $\int_1^3 \frac{1}{18}(11-2x)\mathrm{d}x$ <u>or</u> area of trapezium	M1	1.1a
	$= \left[\frac{1}{18}(11x - x^2)\right]_1^3$		
	$= \frac{7}{9}$	A1	1.1b
		(2)	
(b)	Since P(X < 3) > 0.75, the upper quartile is less than 3	B1ft	2.2a
		(1)	
(c)	$\mathrm{E}(X^2) = \int_1^4 \frac{1}{18}x^2(11-2x)\mathrm{d}x \left[= \frac{23}{4}\right]$	M1	1.1b
	$\mathrm{Var}(X) = \frac{23}{4} - \left(\frac{9}{4}\right)^2$	M1	1.1b
	$= \frac{11}{16}$	A1	1.1b
		(3)	
(d)	F(4) = 1 → $\frac{1}{18}(11(4) - 4^2 + c) = 1$ <u>or</u> F(1) = 0 → $\frac{1}{18}(11(1) - 1^2 + c) = 0$	M1	2.1
	c = –10 *	A1*cso	1.1b
		(2)	
(e)	F(m) = 0.5	M1	1.2
	$\frac{1}{18}(11m - m^2 - 10) = 0.5 \rightarrow m^2 - 11m + 19 = 0$ and attempt to solve	M1	1.1b
	$m = \frac{11 \pm \sqrt{11^2 - 4(19)}}{2}$ [= 2.1458 or 8.8541…]		
	m = 2.1458… **<u>2.15</u>** (only)	A1	2.2a
		(3)	
		(11 marks)	

Notes:

(a)

M1: For integrating f(x) with correct limits **or** for finding area of trapezium

A1: For $\frac{7}{9}$ (allow awrt 0.778)

(b)

B1ft: For comparison of their (a) with 0.75 and concluding that the upper quartile is less than 3

(c)

M1: For an attempt to find $\mathrm{E}(X^2)$

M1: For use of $\mathrm{Var}(X) = \mathrm{E}(X^2) - \left(\frac{9}{4}\right)^2$

A1: For $\frac{11}{16}$ (allow awrt 0.688)(M1 marks may be implied by a correct answer)

Question 6 notes continued:
(d) **M1:** For use of F(4) = 1 or F(1) = 0 **A1*cso:** For a fully correct solution leading to given answer with no errors seen
(e) **M1:** For use of F(m) = 0.5 **M1:** For setting up quadratic and attempt to solve **A1:** For 2.15 and rejecting the other solution

Question	Scheme	Marks	AOs
7(a)	$r = \dfrac{284.4 - \frac{251(12)}{10}}{\sqrt{10.36 \times 40.9}}$	M1	1.1b
	$r = -0.79671...$ awrt **<u>–0.797</u>**	A1	1.1b
		(2)	
(b)	$b = \dfrac{'-16.4'}{10.36}$	M1	3.3
	$a = \dfrac{251}{10} - 'b'\dfrac{12}{10}$	M1	1.1b
	$y = 27.0 - 1.58x$	A1	1.1b
		(3)	
(c)	$y = [27.0 - 1.58(2)] = 23.84$ awrt **<u>23.8</u>**	B1ft	3.4
		(1)	
(d)	$\text{RSS} = 40.9 - \dfrac{(-16.4)^2}{10.36}$	M1	1.1b
	$\text{RSS} = 14.938...$ awrt **<u>14.9</u>**	A1	1.1b
		(2)	
(e)	$\sum$residuals = 0 → –0.63 + (–0.32) + … + f + (–1.88) = 0	M1	3.1a
	f = **<u>–1.04</u>**	A1	1.1b
		(2)	
(f)	The residuals should be randomly scattered above and below zero so linear model may not be appropriate	B1	3.5b
		(1)	
			(11 marks)

Notes:

(a)
M1: For a complete correct method for finding r
A1: For awrt –0.797

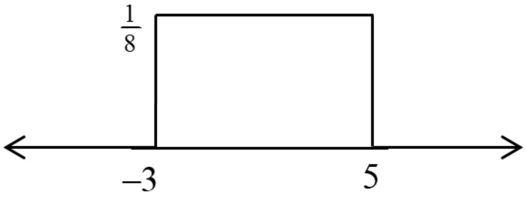
(b)
M1: For use of a correct model i.e. a correct expression for b (ft their S_{xy})
M1: For use of a correct model i.e. a correct (ft) expression for a
A1: For $y = 27.0 - 1.58x$ [a correct answer here can imply both method marks]

(c)
B1: For awrt 23.8 (evaluating their model found in part (b) with $x = 2$)

(d)
M1: For a correct expression for RSS
A1: For awrt 14.9

(e)
M1: For use of $\sum$residuals = 0 [Use of regression equation needs correct sign]
A1: For –1.04

(f)
B1: For identifying that the residuals are not randomly scattered above and below zero and concluding the linear regression model may not be appropriate

Question	Scheme	Marks	AOs
8(a)		B1 (shape) B1 (labels)	1.1b 1.1b
		(2)	
(b)	$\mathrm{P}(X < 2(k - X)) = \mathrm{P}(X < \tfrac{2}{3}k)$	M1	3.1a
	$\dfrac{\tfrac{2}{3}k - (-3)}{5 - (-3)} = 0.25$	M1	1.1b
	$k = -\dfrac{3}{2}$	A1	1.1b
		(3)	
(c)	$\mathrm{E}(X^3) = \displaystyle\int_{-3}^{5} \frac{1}{5-(-3)} x^3 \mathrm{d}x$	M1	2.1
	$= \left[\dfrac{1}{32}x^4\right]_{-3}^{5} = \tfrac{1}{32}(5^4 - (-3)^4)$	dM1	1.1b
	$=17^*$	A1*cso	1.1b
		(3)	
			(8 marks)

Notes:

(a)
B1: For correct shape
B1: For correct labels

(b)
M1: For simplifying to $\mathrm{P}(X < \tfrac{2}{3}k)$
M1: For equating probability expression to 0.25
A1: For $-\dfrac{3}{2}$

Another method for part (b) is:
M1: For understanding $2[k - x] = -1$ and $x = -1$
M1: For substitution and attempt to solve
A1: For $-\dfrac{3}{2}$

(c)
B1: For integrating $x^3\,\mathrm{f}(x)$
M1: For use of correct limits (dependent on previous M1)
A1*: For fully correct solution leading to the given answer with no errors seen

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2H: Further Mechanics 1 and Decision Mathematics 1

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2H

You must have:
Decision Mathematics 1 question insert for Section B
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The questions for Section B (Decision Mathematics) can be found in the question insert.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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S 5 8 5 3 4 A 0 1 1 8

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SECTION A

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 1 continued

(Total for Question 1 is 10 marks)

2. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3. [*In this question use* $g = 10\,\text{m}\,\text{s}^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\,\text{m}\,\text{s}^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\text{m}\,\text{s}^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\text{m}\,\text{s}^{-1}$.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 8 marks)

4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \dfrac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \dfrac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

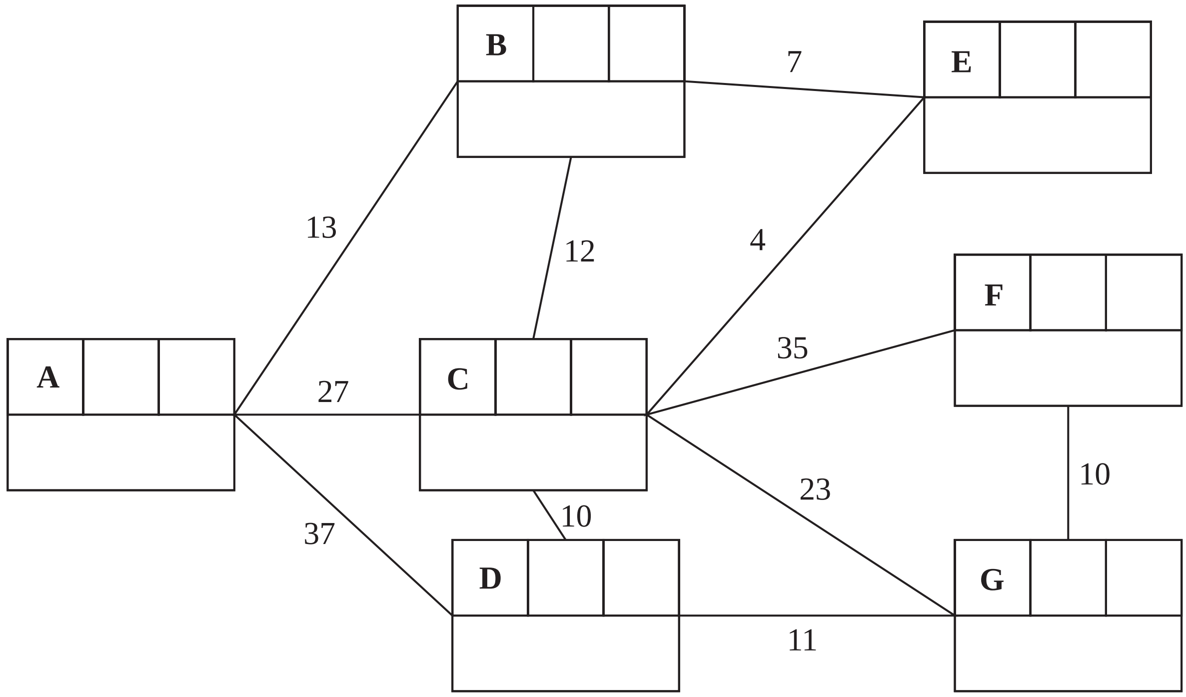
(Total for Question 4 is 16 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

The questions for this section, Decision Mathematics 1, are provided in the Decision Mathematics 1 question insert.

5.



Key:

Vertex	Order of labelling	Final value
Working values		

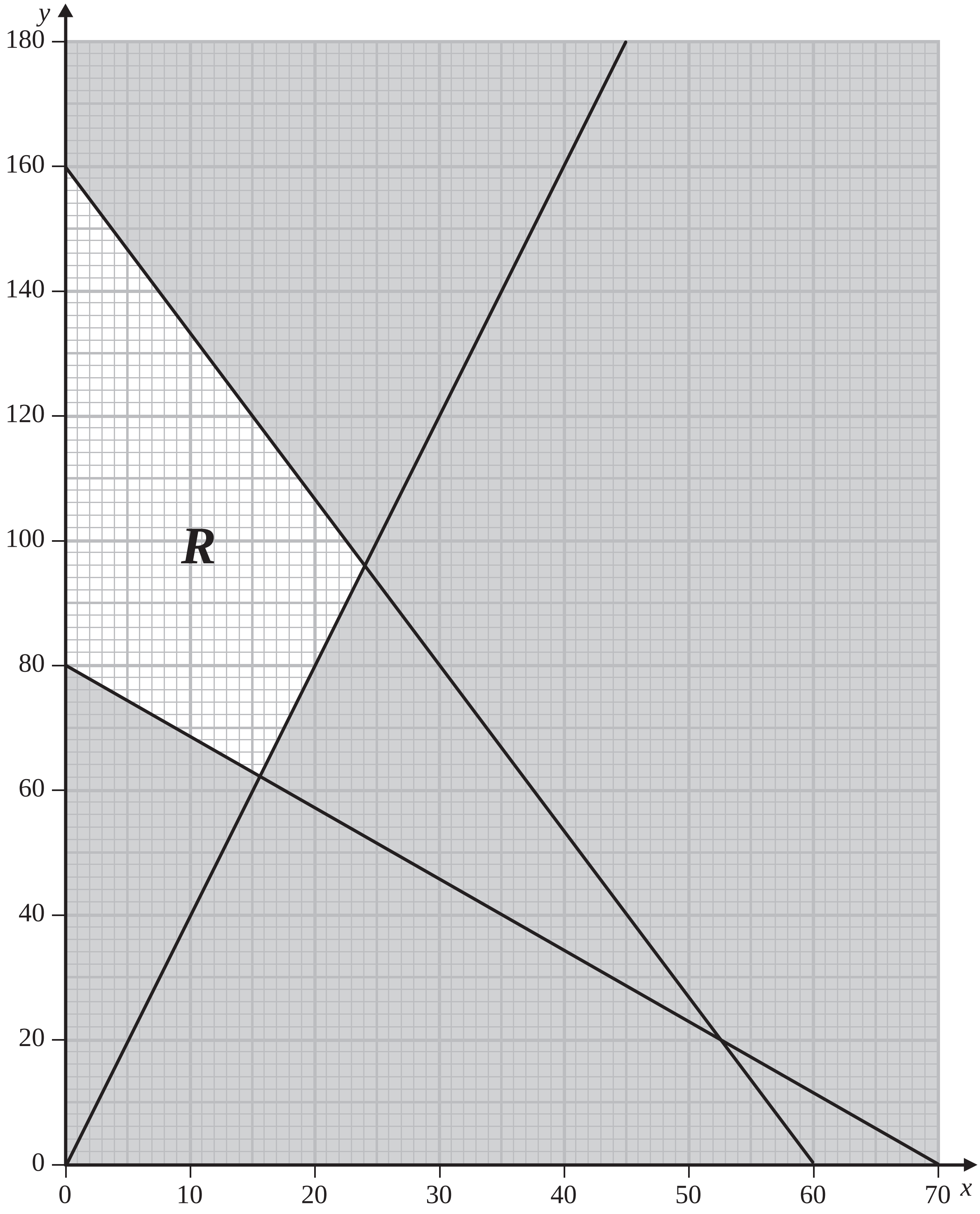
Shortest path: ____________________

Length of shortest path: ____________________

Question 5 continued

(Total for Question 5 is 12 marks)

6.



Question 6 continued

(Total for Question 6 is 7 marks)

7. (a) and (b)

(Total for Question 7 is 7 marks)

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8.

(Total for Question 8 is 9 marks)

9.

(Total for Question 9 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2H: Section B Decision Mathematics 1

Sample Assessment Material for first teaching September 2017

Paper Reference
8FM0/2H

Decision Mathematics 1 question insert for Section B
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SECTION B

Answer ALL questions. Write your answers in the answer book provided.

5.

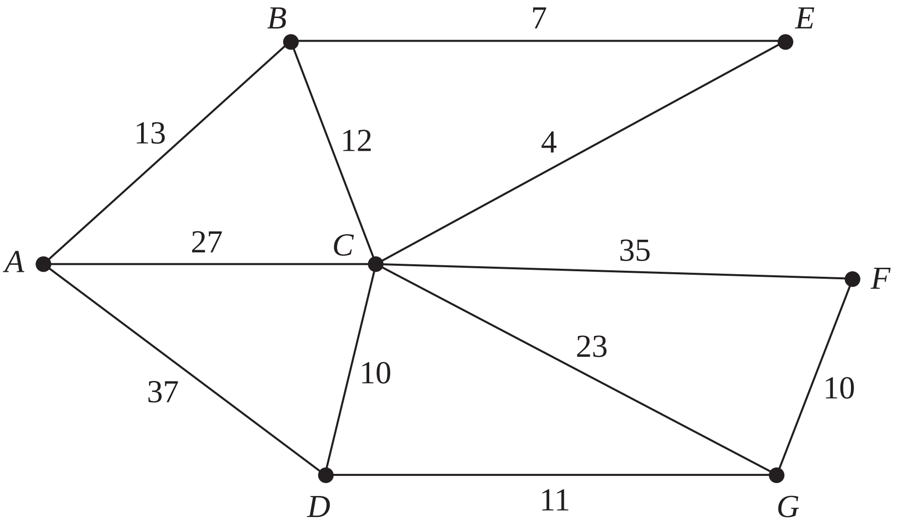


Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from A to F. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at A, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, BG, is added to the network. A route of minimum length that traverses each pipe, including BG, needs to be found. The route must start and finish at A.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of BG.

(d) Calculate the length of BG. You must show your working.

(2)

(Total for Question 5 is 12 marks)

6.

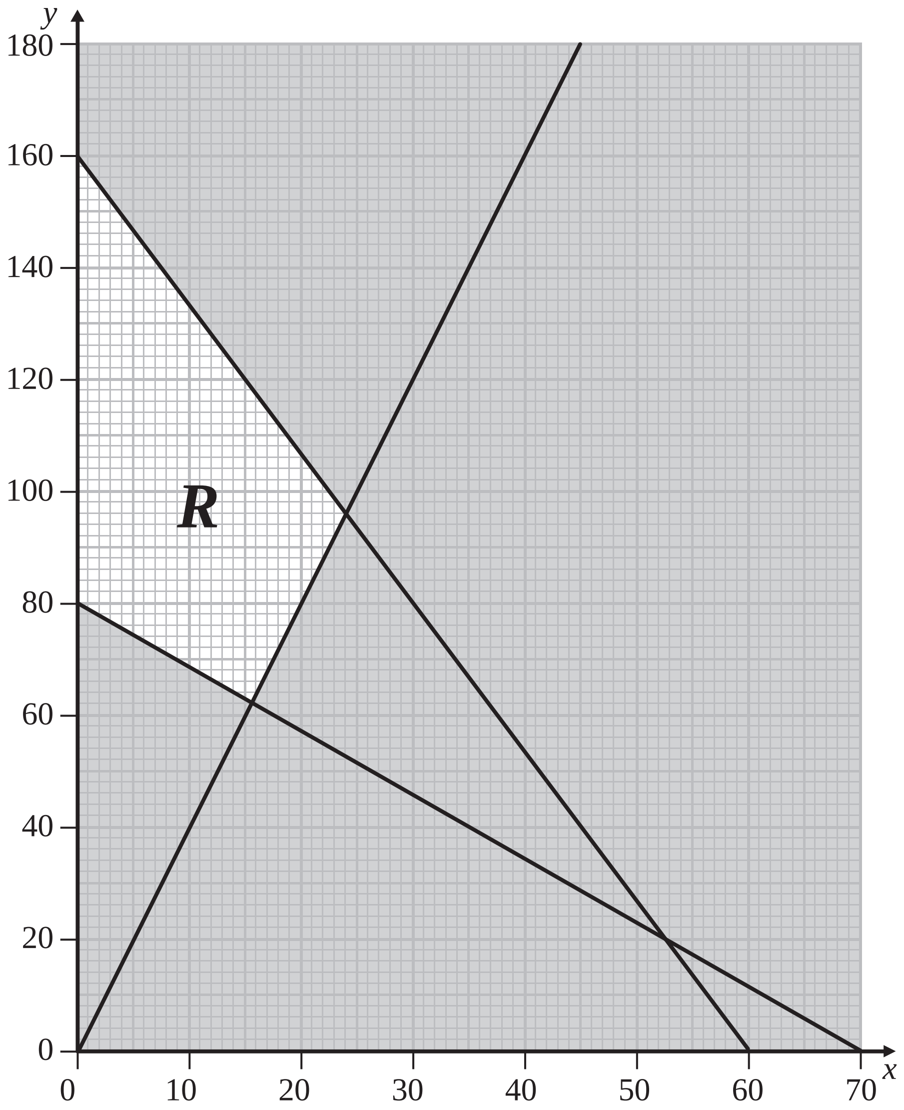


Figure 2

A teacher buys pens and pencils. The number of pens, *x*, and the number of pencils, *y*, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leqslant 480$$

$$8x + 7y \geqslant 560$$

$$y \geqslant 4x$$

$$x, y \geqslant 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 6 is 7 marks)

7.

Activity	Time taken (days)	Immediately preceding activities
A	5	-
B	7	-
C	3	-
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) (i) Show that the project can be completed in 21 days, showing your working.

(ii) Identify the critical activities.

(4)

(Total for Question 7 is 7 marks)

8. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1$, 2^{x}, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 8 is 9 marks)

9. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 9 is 5 marks)

TOTAL FOR SECTION B IS 40 MARKS

Paper 2 Option **H**

Further Mechanics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g \quad \Rightarrow \quad v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1*	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh \Rightarrow h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		(1)	
			(10 marks)

Notes:

(a)
M1: For a complete method to find v
A1: For a correct value (may be numerical)
M1: For a complete method to find u
A1: For a correct value (may be numerical)
M1: For finding both v and u and use of Newton's Law of Restitution
A1*: For the given answer

(b)
M1: For use of Newton's Law of Restitution to find rebound speed
M1: For a complete method to find h
A1: For 0.15 (m) oe

(c)
B1: For a clear description including reference to a quarter

Question	Scheme	Marks	AOs
2(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5\,g \times 20$	A1	1.1b
	= 58.25 = 58 (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20\,R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1ft	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)
M1: For a difference in KE and PE
A1: For a correct expression
A1: For either 58 (2sf) or 58.3(3sf)

(b)
M1: For use of work-energy principle
A1ft: For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

(c)
B1: For variable resistance oe

Question	Scheme	Marks	AOs
3(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force × Speed = 30 × 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force × Speed = 70 × 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)

B1: For force = 30 seen

M1: For use of $P = Fv$

A1ft: For 120 (W), follow through on their '30'

(b)

M1: For resolving parallel to the slope with correct no. of terms and $60g$ resolved

A1: For a correct equation

A1: For $F = 70$

M1: For use of $P = Fv$

A1ft: For 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
4(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
		(16 marks)	

Notes:

(a)
M1: For use of CLM, with correct no. of terms, condone sign errors
A1: For a correct equation
M1: For use of Newton's Law of Restitution, with e on the correct side
A1: For a correct equation
M1: For setting up *two* equations and solving their equations for v
A1: For a correct expression for v

(b)
M1: For use of an appropriate inequality
A1: For a complete range of values of e

(c)
M1: For solving their equations for w
A1: For the given answer

Question 4 notes continued:

(d)

M1: For substituting $e = \frac{5}{9}$ into their v and w

A1: For correct expressions for v and w

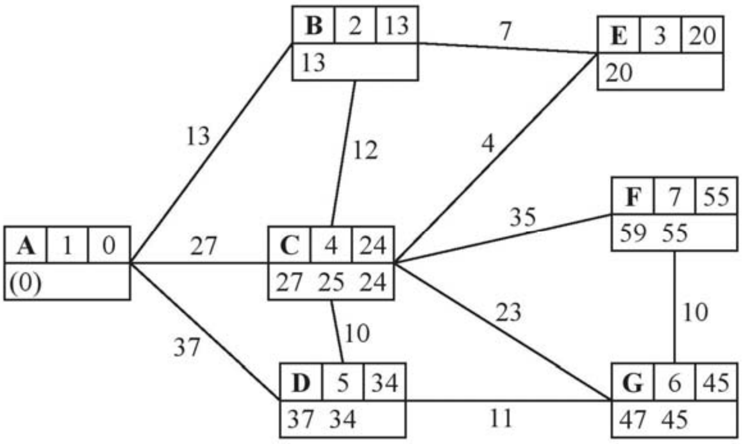
M1: For use of Newton's Law of Restitution, with e on the correct side

M1: For use of appropriate inequality

A1: For a correct inequality

A1: For a correct range

Decision Mathematics 1 Mark Scheme (Section **B**)

Question	Scheme	Marks	AOs
5(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
		(1)	
(d)	$189+x+34=213+2x$	M1	3.1b
	$x=10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)
M1: For a larger number replaced by a smaller one in the working values boxes at C, D, F or G
A1: For all values correct (and in correct order) at A, B, C and D
A1: For all values correct (and in correct order) at E, F & G
A1: For the correct path
A1ft: For 55 or ft their final value at F

(b)
M1: For 3 correct pairings of the four odd nodes (A,B, D & G)
A1: At least two pairings and totals correct
A2: All three pairings and totals correct
A3ft: Selecting their shortest pairing, and stating that these arcs should be repeated

Question **5** notes continued:
(c) **B1ft:** For 213 or 189 + their shortest repeat **M1:** For translating the information in the question in to an equation involving x, $2x$ and 34 **A1:** For a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
6	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15 ± 1, 63 ± 2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and y = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
			(7 marks)

Notes:

M1: Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C

M1: Solving simultaneous equations

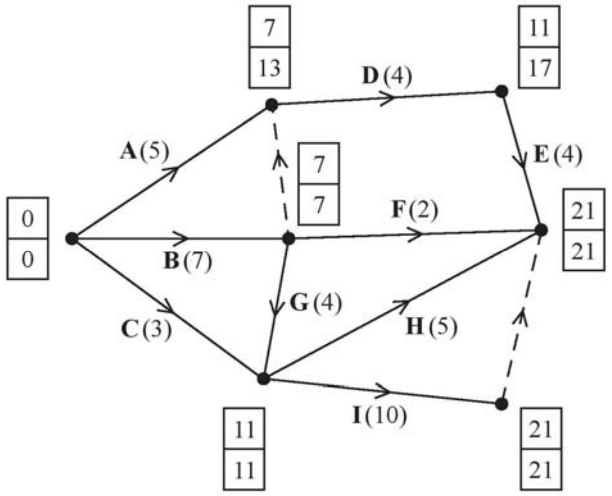
A1: cao

M1: Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$

M1: Testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C

A1: cao – deducing from tests which integer solution is both valid and optimal

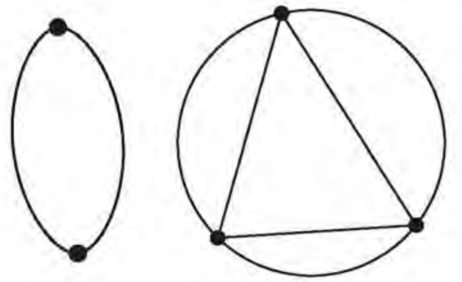
A1ft: Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
7(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
		M1 A1	2.1 1.1b
	The number(s) at the end of activity E indicate this project can be completed in 21 days	A1ft	2.2a
	Critical activities: B, G, I	A1	1.1b
		(4)	
			(7 marks)

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: All boxes completed, number generally increasing L to R (condone one "rogue")
A1: All values cao
A1: Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes)
A1: Critical activities correct

Question	Scheme	Marks	AOs
8(a)	e.g. a graph cannot contain an odd number of odd nodes e.g. number of arcs $=\frac{1+3+4+4+5}{2}=8.5\notin\mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+\left(x+1\right)+\left(2^{x+1}-3\right)+\left(11-x\right)=2\left(18\right)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0\Rightarrow x=\ldots$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0\Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1	2.4
		A1	2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
			(9 marks)

Notes:

(a)

B1: Explanation referring to need for an even number of odd nodes oe

(b)

M1: Forming an equation involving the orders of the 5 odd nodes and $2(18)$

M1: Simplifies to a quadratic in 2^x and attempts to solve

A1: 2 cao

M1: Construct an argument involving the order of the 5 nodes

A1: Explanation considering the number of odd nodes

A1: Deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)

M1: Interprets mathematical language to construct a disconnected graph

A1: Deduce a correct graph

Question	Scheme	Marks	AOs
9	Minimise $(C=)\ 25x+35y$	B1	3.3
	Subject to: $(500x+800y\geqslant 150\ 000\Rightarrow)\ 5x+8y\geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y)\leqslant x\leqslant\frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y\leqslant 13x$ **and** $13y\geqslant 7x$ $x, y\geqslant 0$	A1	1.1b
			(5 marks)

Notes:

B1: A correct objective function + minimise
B1: Translate information in to a correct inequality
M1: For translating the information given into the LHS inequality
M1: For translating the information given in to the RHS inequality
A1: Simplifying to the correct inequalities

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2J: Further Mechanics 1 and Further Mechanics 2

Sample Assessment Material for first teaching September 2017
Time: 1 hour 40 minutes

Paper Reference
8FM0/2J

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

(6)

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

Question 1 continued

(Total for Question 1 is 10 marks)

2. A small stone of mass 0.5 kg is thrown vertically upwards from a point *A* with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point *B* which is 20 m vertically above the point *A*. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from *A* to *B*.

(3)

The air resistance is modelled as a constant force of magnitude *R* newtons.

(b) Find the value of *R*.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3. [*In this question use g* $=10\,\text{m}\,\text{s}^{-2}$]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\,\text{m}\,\text{s}^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\text{m}\,\text{s}^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\text{m}\,\text{s}^{-1}$.

(5)

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DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 8 marks)

4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \frac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 4 continued

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 16 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

5. A particle P moves on the x-axis. At time t seconds the velocity of P is $v\,\mathrm{m\,s^{-1}}$ in the direction of x increasing, where

$$v = (t - 2)(3t - 10), \qquad t \geqslant 0$$

When $t = 0$, P is at the origin O.

(a) Find the acceleration of P at time t seconds.

(2)

(b) Find the total distance travelled by P in the first 2 seconds of its motion.

(3)

(c) Show that P never returns to O, explaining your reasoning.

(3)

Question 5 continued

(Total for Question 5 is 8 marks)

6. A light inextensible string has length $7a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 5a$. A particle of mass m is attached to a point P on the string where $AP = 4a$. The particle moves in a horizontal circle with constant angular speed ω, with both AP and BP taut.

(a) Show that

(i) the tension in AP is $\dfrac{4m}{25}(9a\omega^2 + 5g)$

(ii) the tension in BP is $\dfrac{3m}{25}(16a\omega^2 - 5g)$.

(10)

The string will break if the tension in it reaches a magnitude of $4mg$.

The time for the particle to make one revolution is S.

(b) Show that

$$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$$

(5)

(c) State how in your calculations you have used the assumption that the string is light.

(1)

Question 6 continued

Question 6 continued

(Total for Question 6 is 16 marks)

7.

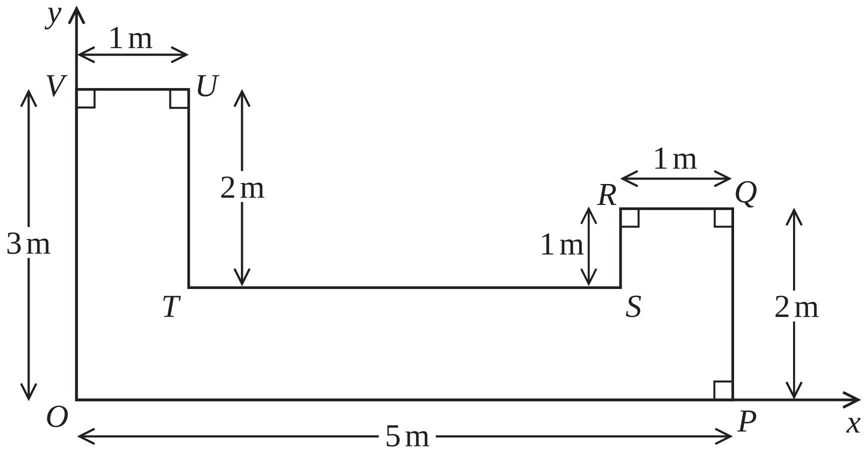

Figure 1

Figure 1 shows the shape and dimensions of a template *OPQRSTUV* made from thin uniform metal.
$OP = 5$ m, $PQ = 2$ m, $QR = 1$ m, $RS = 1$ m, $TU = 2$ m, $UV = 1$ m, $VO = 3$ m.
Figure 1 also shows a coordinate system with O as origin and the x-axis and y-axis along OP and OV respectively. The unit of length on both axes is the metre.

The centre of mass of the template has coordinates $(\bar{x}, \bar{y})$.

(a) (i) Show that $\bar{y} = 1$

(ii) Find the value of $\bar{x}$.

(7)

A new design requires the template to have its centre of mass at the point (2.5, 1). In order to achieve this, two circular discs, each of radius r metres, are removed from the template which is shown in Figure 1, to form a new template L. The centre of the first disc is (0.5, 0.5) and the centre of the second disc is (0.5, a) where a is a constant.

(b) Find the value of r.

(4)

(c) (i) Explain how symmetry can be used to find the value of a.

(ii) Find the value of a.

(2)

The template L is now freely suspended from the point U and hangs in equilibrium.

(d) Find the size of the angle between the line TU and the horizontal.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 16 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Paper 2 Option J

Further Mechanics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g$ => $v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g \times 0.6$ => $u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1*	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh$ => $h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		(1)	
		(10 marks)	

Notes:

(a)
M1: For a complete method to find v
A1: For a correct value (may be numerical)
M1: For a complete method to find u
A1: For a correct value (may be numerical)
M1: For finding both v and u and use of Newton’s Law of Restitution
A1*: For the given answer

(b)
M1: For use of Newton’s Law of Restitution to find rebound speed
M1: For a complete method to find h
A1: For 0.15 (m) oe

(c)
B1: For a clear description including reference to a quarter

Question	Scheme	Marks	AOs
2(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5\,g \times 20$	A1	1.1b
	= 58.25 = 58 (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20\,R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1ft	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
		(6 marks)	

Notes:

(a)
M1: For a difference in KE and PE
A1: For a correct expression
A1: For either 58 (2sf) or 58.3(3sf)

(b)
M1: For use of work-energy principle
A1ft: For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

(c)
B1: For variable resistance oe

Question	Scheme	Marks	AOs
3(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force × Speed = 30 × 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force × Speed = 70 × 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)

B1: For force = 30 seen

M1: For use of $P = Fv$

A1ft: For 120 (W), follow through on their '30'

(b)

M1: For resolving parallel to the slope with correct no. of terms and $60g$ resolved

A1: For a correct equation

A1: For $F = 70$

M1: For use of $P = Fv$

A1ft: For 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
4(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in *u* and *v* and solving for *v*	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for *w*	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
		(16 marks)	

Notes:

(a)
M1: For use of CLM, with correct no. of terms, condone sign errors
A1: For a correct equation
M1: For use of Newton's Law of Restitution, with *e* on the correct side
A1: For a correct equation
M1: For setting up *two* equations and solving their equations for *v*
A1: For a correct expression for *v*

(b)
M1: For use of an appropriate inequality
A1: For a complete range of values of *e*

(c)
M1: For solving their equations for *w*
A1: For the given answer

Question 4 notes continued:

(d)

M1: For substituting $e = \frac{5}{9}$ into their v and w

A1: For correct expressions for v and w

M1: For use of Newton’s Law of Restitution, with e on the correct side

M1: For use of appropriate inequality

A1: For a correct inequality

A1: For a correct range

Further Mechanics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5 (a)	Multiply out and differentiate wrt *t*	M1	1.1b
	$v = 3t^2 - 16t + 20 \Rightarrow a = 6t - 16$	A1	1.1b
		(2)	
(b)	Multiply out and integrate wrt *t*	M1	1.1b
	$s = \int 3t^2 - 16t + 20\mathrm{d}t = t^3 - 8t^2 + 20t(+C)$	A1	1.1b
	$t = 0, s = 0$ => $C = 0$ $t = 2, s = 8 - 32 + 40 = 16$	A1	1.1b
		(3)	
(c)	$s = 0 \Rightarrow t^3 - 8t^2 + 20t = 0$ and $t \neq 0 \Rightarrow t^2 - 8t + 20 = 0$	M1	2.1
	Explanation to show that $t^2 - 8t + 20 > 0$ for all *t*.	M1	2.4
	So $s = 0$ has no non-zero solutions, so *s* is never zero again, so never returns to *O* *	A1*	3.2a
		(3)	
			(8 marks)

Notes:

(a)
M1: For multiplying out and differentiating (powers decreasing by 1)
A1: For a correct expression for *a*

(b)
M1: For multiplying out and integrating (powers increasing by 1)
A1: For a correct expression for *s* with or without *C*
A1: For *C* = 0 and correct final answer

(c)
M1: For equating their *s* to 0 and producing a quadratic
M1: For clear explanation that $t^2 - 8t + 20 > 0$ for all *t* (e.g. completing the square or another complete method)
A1*: For a correct conclusion in context

Question	Scheme	Marks	AOs
6(a)	$\cos\alpha = \frac{4}{5}$ or $\sin\alpha = \frac{3}{5}$	B1	1.1b
	$r = 4a\sin\alpha$	B1	1.1b
	Resolving vertically	M1	3.1b
	$T_1\cos\alpha - T_2\sin\alpha = mg$	A1	1.1b
	Resolving horizontally	M1	3.1b
	$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$	A1	1.1b
	$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$	A1	1.1b
	Solving for either tension	M1	2.1
	$T_1 = \frac{4m}{25}(9a\omega^2 + 5g)$ *	A1*	1.1b
	$T_2 = \frac{3m}{25}(16a\omega^2 - 5g)$ *	A1*	1.1b
		(10)	
(b)	$\frac{4m}{25}(9a\omega^2 + 5g) < 4mg$	M1	2.1
	$\frac{3m}{25}(16a\omega^2 - 5g) > 0$	M1	2.1
	$\omega > \sqrt{\frac{5g}{16a}}$ or $\omega < \sqrt{\frac{20g}{9a}}$	A1	2.2a
	$S = \frac{2\pi}{\omega}$	M1	1.1b
	$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$ *	A1*	1.1b
		(5)	
(c)	String being light implies that the tension is constant in both portions of the string	B1	3.5b
		(1)	
			(16 marks)

Notes:

(a)

B1: For correct trig. ratio seen

B1: For a correct radius expression seen

M1: For resolving vertically with correct no. of terms and tensions resolved

A1: For a correct equation

M1: For resolving horizontally with correct no. of terms and tensions resolved

A1A1: For a correct equation

M1: For solving their two equations to find either tension

A1*: For the given answer

A1*: For the given answer

Question 6 notes continued:

(b)

M1: For use of $T_1 < 4mg$

M1: For using $T_2 > 0$

A1: For a correct inequality (either) for ω

M1: For use of $S = \dfrac{2\pi}{\omega}$ with either critical value

A1*: For given answer

(c)

B1: For a clear explanation

Question	Scheme	Marks	AOs
7(a)	Rel. Mass: 2 5 1 8	B1	1.2
	y : 2 0.5 1.5 $\bar{y}$	B1	1.2
	x : 0.5 2.5 4.5 $\bar{x}$	B1	1.2
	$(2 \times 2) + (5 \times 0.5) + (1 \times 1.5) = 8\bar{y}$	M1	2.1
	$\bar{y} = 1$ *	A1*	1.1b
	$(2 \times 0.5) + (5 \times 2.5) + (1 \times 4.5) = 8\bar{x}$	M1	2.1
	$\bar{x} = 2.25$	A1	1.1b
		(7)	
(b)	Use of correct strategy to solve the problem by use of 'moments equation'	M1	3.1b
	$(8 \times 2.25) - (2\pi r^2 \times 0.5) = (8 - 2\pi r^2)2.5$	A1ft	1.1b
	Solving for r	M1	1.1b
	$r = \frac{1}{\sqrt{2\pi}} = 0.399$	A1	1.1b
		(4)	
(c)	Since $\bar{y}$ for original plate is 1, holes must be symmetrically placed about the line $y = 1$	B1	2.4
	$a = 1.5$	B1	2.2a
		(2)	
(d)	Use of tan from an appropriate triangle	M1	1.1a
	$\tan\alpha = \frac{2}{1.5} = \frac{4}{3}$	A1ft	1.1b
	$\alpha = 53.1°$	A1	1.1b
		(3)	
		(16 marks)	

Notes:

(a)
B1: For correct relative masses
B1: For correct y values
B1: For correct x values
M1: For a moments equation, correct no. of terms, condone sign errors
A1*: For a correct given answer (1)
M1: For a moments equation, correct no. of terms
A1: For 2.25

(b)
M1: For a moments equation, correct no. of terms, condone sign errors
A1ft: For a correct equation, follow through on their $\bar{x}$
M1: For solving for r
A1: For 0.399 or 0.40

Question 7 notes continued:
(c) **B1:** For consideration of symmetry about $y = 1$ **B1:** For $a = 1.5$
(d) **M1:** For use of tan from an appropriate triangle **A1ft:** For a correct equation, follow through on their a **A1:** For a correct angle

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2K: Decision Mathematics 1 and Decision Mathematics 2

Sample Assessment Material for first teaching September 2017

Time: 1 hour 40 minutes

Paper Reference

8FM0/2K

You must have:

Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If a pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.
- Do not return this question paper with the answer book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

SECTION A

Answer ALL questions. Write your answers in the answer book provided.

1.

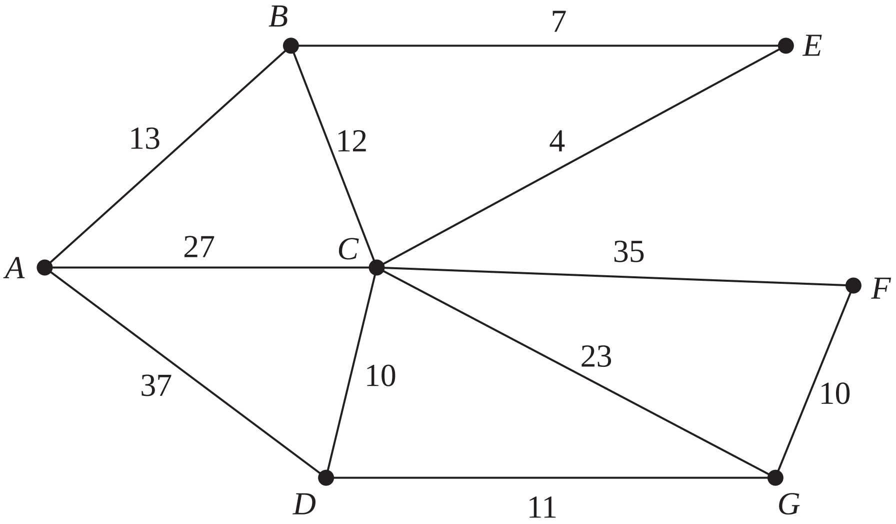

Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from *A* to *F*. State the path and its length.
(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at *A*, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.
(4)

(c) State the minimum length of Gabriel's route.
(1)

A new pipe, *BG*, is added to the network. A route of minimum length that traverses each pipe, including *BG*, needs to be found. The route must start and finish at *A*.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of *BG*.

(d) Calculate the length of *BG*. You must show your working.
(2)

(Total for Question 1 is 12 marks)

2.

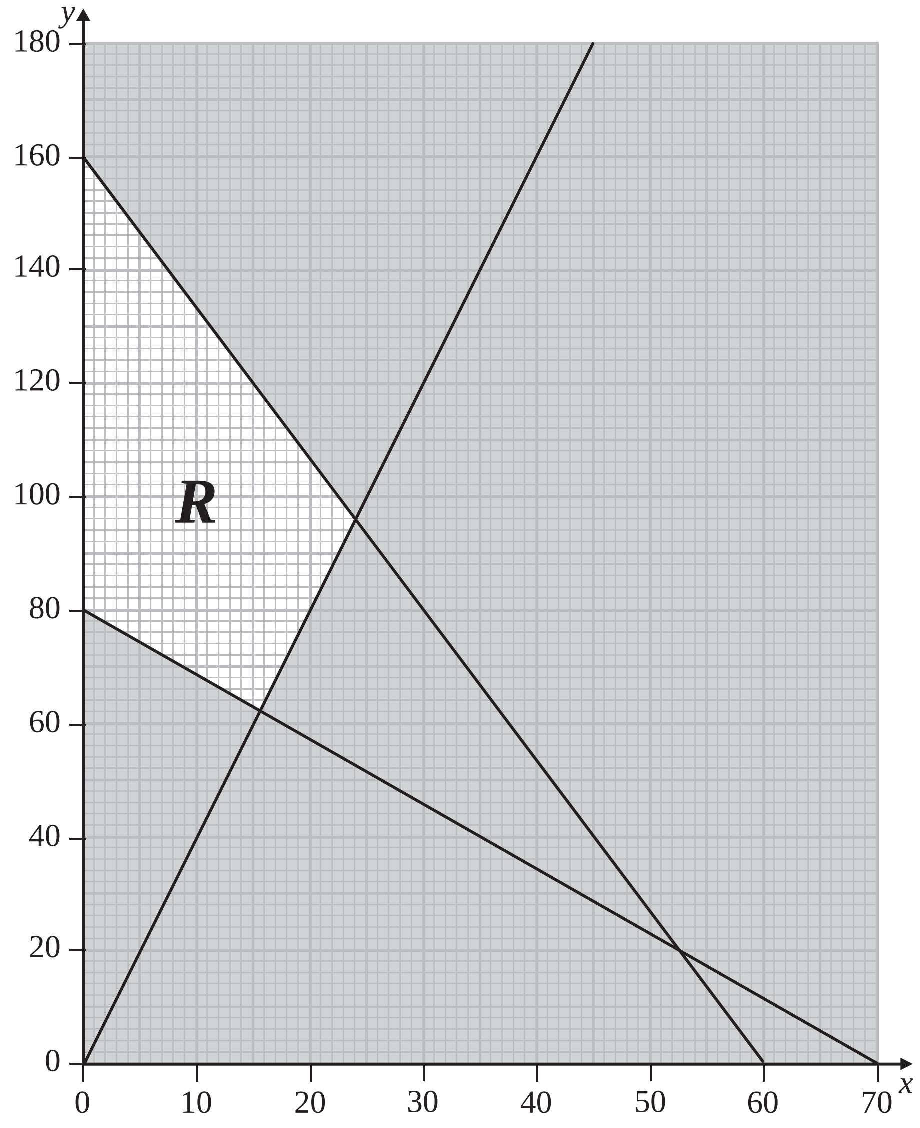

Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leqslant 480$$

$$8x + 7y \geqslant 560$$

$$y \geqslant 4x$$

$$x, y \geqslant 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 2 is 7 marks)

3.

Activity	Time taken (days)	Immediately preceding activities
A	5	–
B	7	–
C	3	–
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) (i) Show that the project can be completed in 21 days, showing your working.

(ii) Identify the critical activities.

(4)

(Total for Question 3 is 7 marks)

4. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1$, 2^x, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

(ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for Question 4 is 9 marks)

5. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let x and y represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 5 is 5 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the answer book provided.

6. Six workers, A, B, C, D, E and F, are to be assigned to five tasks, P, Q, R, S and T.

Each worker can be assigned to at most one task and each task must be done by just one worker.

The time, in minutes, that each worker takes to complete each task is shown in the table below.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

Reducing rows first, use the Hungarian algorithm to obtain an allocation which minimises the total time. You must explain your method and show the table after each stage.

(Total for Question 6 is 9 marks)

7. In two-dimensional space, lines divide a plane into a number of different regions.

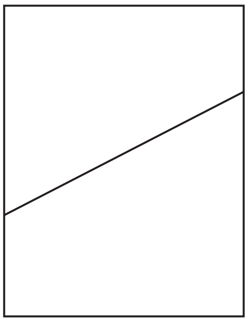

Figure 1

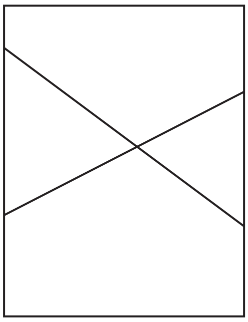

Figure 2

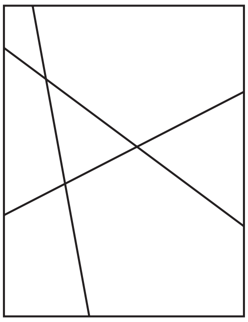

Figure 3

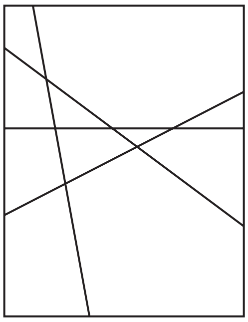

Figure 4

It is known that:

- One line divides a plane into 2 regions, as shown in Figure 1
- Two lines divide a plane into a maximum of 4 regions, as shown in Figure 2
- Three lines divide a plane into a maximum of 7 regions, as shown in Figure 3
- Four lines divide a plane into a maximum of 11 regions, as shown in Figure 4

(a) Complete the table in the answer book to show the maximum number of regions when five, six and seven lines divide a plane.

(1)

(b) Find, in terms of u_n, the recurrence relation for u_{n+1}, the maximum number of regions when a plane is divided by $(n + 1)$ lines where $n \geqslant 1$

(1)

(c) (i) Solve the recurrence relation for u_n

(ii) Hence determine the maximum number of regions created when 200 lines divide a plane.

(3)

(Total for Question 7 is 5 marks)

8.

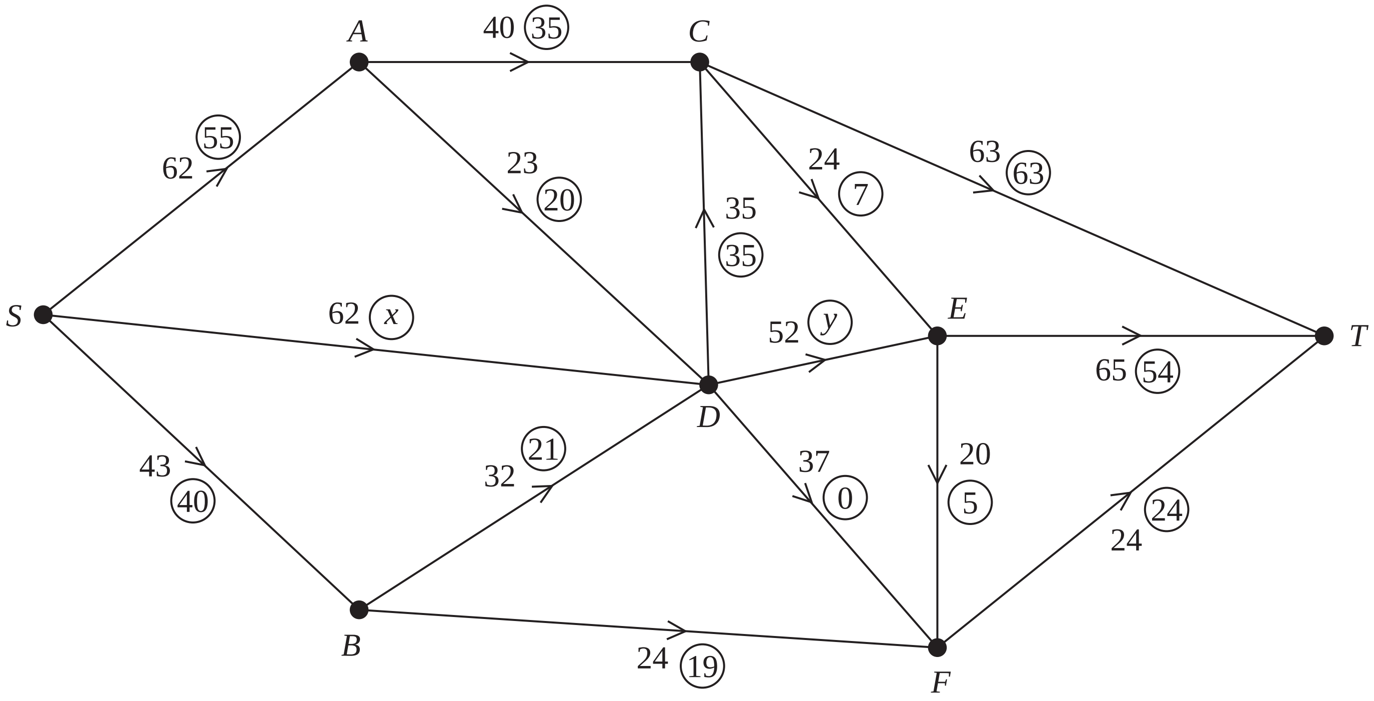

Figure 5

Figure 5 represents a network of corridors in a school. The number on each arc represents the maximum number of students, per minute, that may pass along each corridor at any one time. At 11 am on Friday morning, all students leave the hall (*S*) after assembly and travel to the cybercafé (*T*). The numbers in circles represent the initial flow of students recorded at 11 am one Friday.

(a) State an assumption that has been made about the corridors in order for this situation to be modelled by a directed network.

(1)

(b) Find the value of *x* and the value of *y*, explaining your reasoning.

(3)

Five new students also attend the assembly in the hall the following Friday. They too need to travel to the cybercafé at 11 am. They wish to travel together so that they do not get lost. You may assume that the initial flow of students through the network is the same as that shown in Figure 5 above.

(c) (i) List all the flow augmenting routes from *S* to *T* that increase the flow by at least 5

(ii) State which route the new students should take, giving a reason for your answer.

(3)

(d) Use the answer to part (c) to find a maximum flow pattern for this network and draw it on Diagram 1 in the answer book.

(1)

(e) Prove that the answer to part (d) is optimal.

(3)

The school is intending to increase the number of students it takes but has been informed it cannot do so until it improves the flow of students at peak times. The school can widen corridors to increase their capacity, but can only afford to widen one corridor in the coming term.

(f) State, explaining your reasoning,

(i) which corridor they should widen,

(ii) the resulting increase of flow through the network.

(3)

(Total for Question 8 is 14 marks)

9. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	***B* plays 1**	***B* plays 2**	***B* plays 3**
***A* plays 1**	4	1	2
***A* plays 2**	2	4	3

(a) Verify that there is no stable solution.

(3)

(b) (i) Find the best strategy for player *A*.

(ii) Find the value of the game to her.

(9)

(Total for Question 9 is 12 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Write your name here

Surname	Other names

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Centre Number	Candidate Number
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Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2K: Decision Mathematics 1 and Decision Mathematics 2

Sample Assessment Material for first teaching September 2017

Paper Reference
8FM0/2K

Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over ▶

S58536A

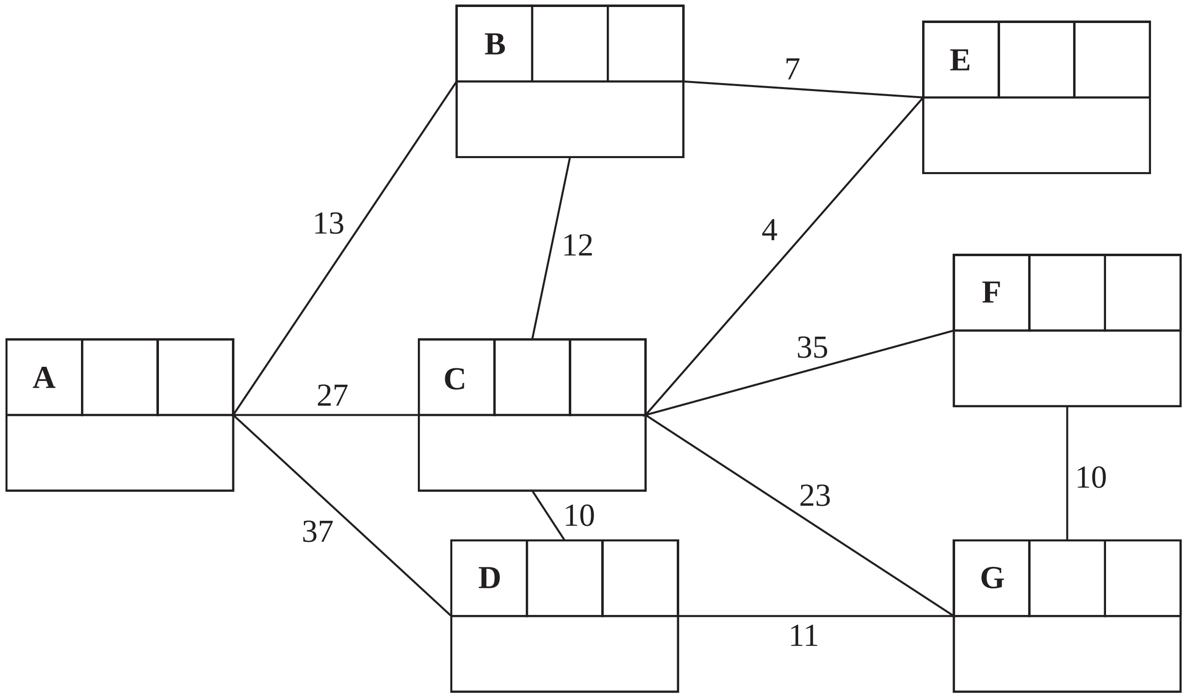
1/1/1/1/1/

S 5 8 5 3 6 A 0 1 1 5

Pearson

SECTION A

1.



Key:

Vertex	Order of labelling	Final values
Working value		

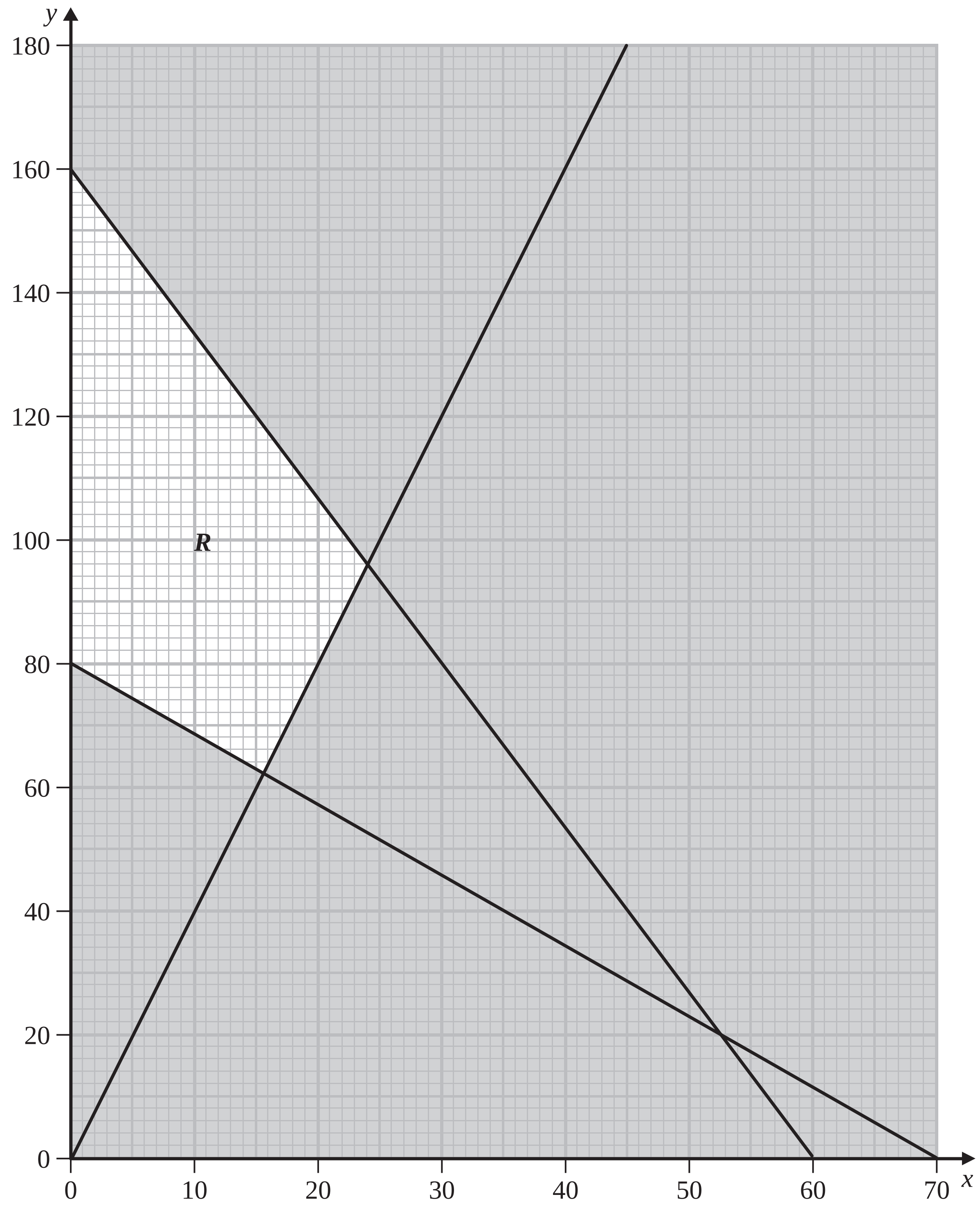
Shortest path: ____________________________

Length of shortest path: ____________________________

Question 1 continued

(Total for Question 1 is 12 marks)

2.



Question 2 continued

(Total for Question 2 is 7 marks)

3. (a) and (b)

(Total for Question 3 is 7 marks)

4.

(Total for Question 4 is 9 marks)

5.

(Total for Question 5 is 5 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

6.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

	P	Q	R	S	T	
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	P	Q	R	S	T	
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	P	Q	R	S	T	
A						
B						
C						
D						
E						
F						

(Total for Question 6 is 9 marks)

7. (a)

Number of lines	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Maximum number of regions	2	4	7	11			

(Total for Question 7 is 5 marks)

8.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

(d)

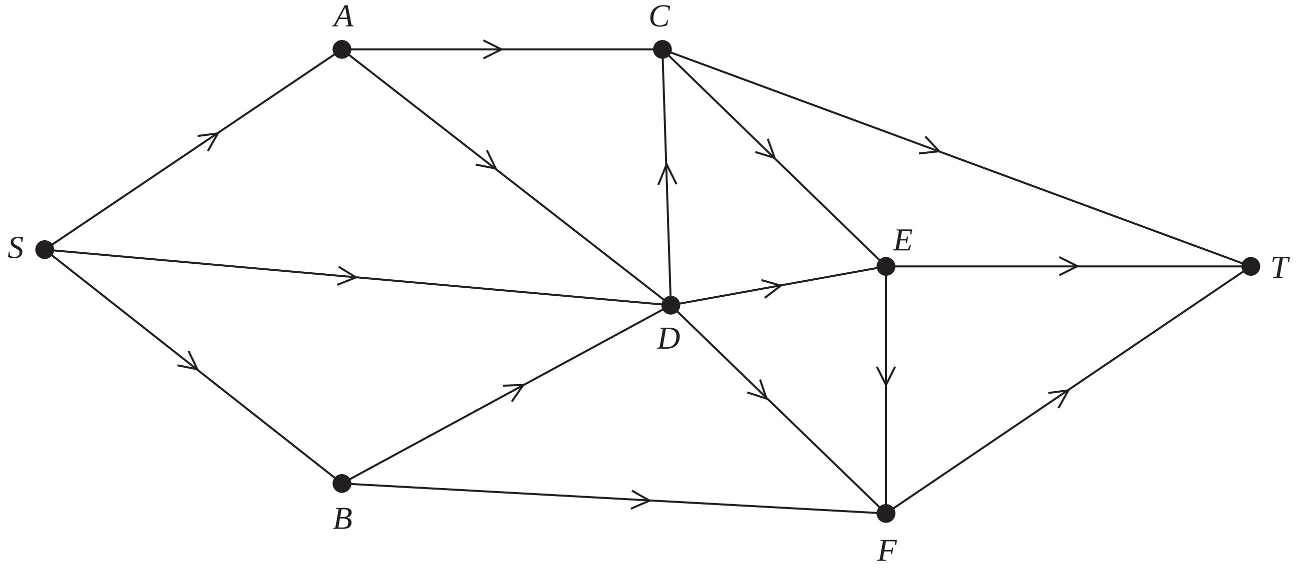


Diagram 1

(Total for Question 8 is 14 marks)

9.

	***B* plays 1**	***B* plays 2**	***B* plays 3**
***A* plays 1**	4	1	2
***A* plays 2**	2	4	3

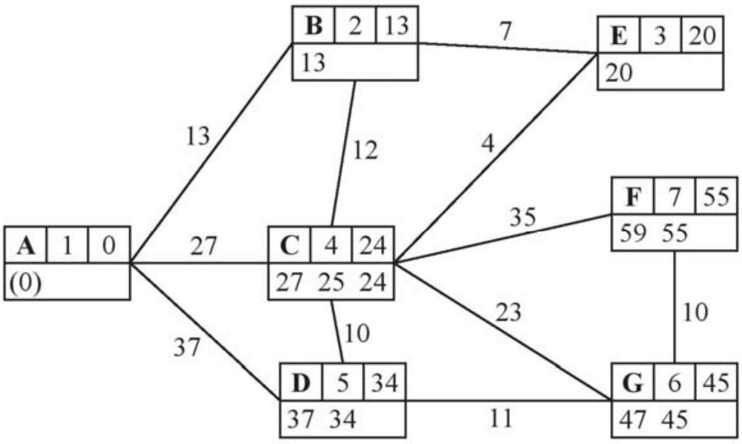
Question 9 continued

(Total for Question 9 is 12 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Paper 2 Option K

Decision Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
		(1)	
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)

M1: For a larger number replaced by a smaller one in the working values boxes at C, D, F or G

A1: For all values correct (and in correct order) at A, B, C and D

A1: For all values correct (and in correct order) at E, F & G

A1: For the correct path

A1ft: For 55 or ft their final value at F

(b)

M1: For 3 correct pairings of the four odd nodes (A,B, D & G)

A1: At least two pairings and totals correct

A2: All three pairings and totals correct

A3ft: Selecting their shortest pairing, and stating that these arcs should be repeated

Question 1 notes continued:
(c) **B1ft:** For 213 or 189 + their shortest repeat **M1:** For translating the information in the question in to an equation involving x, $2x$ and 34 **A1:** For a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
2	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y =	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from (15 ± 1, 63 ± 2)	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	x = 15 and y = 63	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
			(7 marks)

Notes:

M1: Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C

M1: Solving simultaneous equations

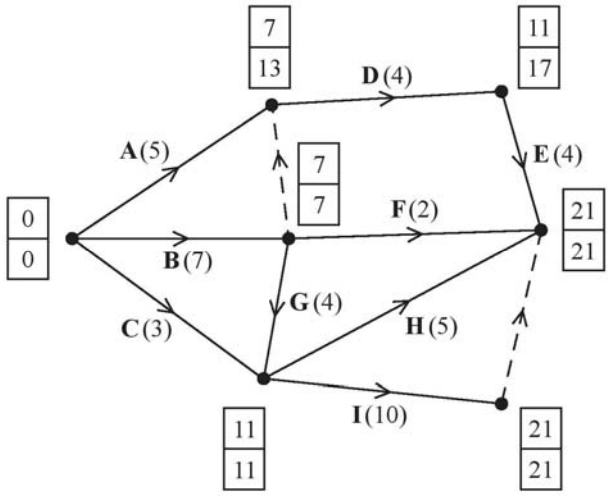
A1: cao

M1: Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$

M1: Testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C

A1: cao – deducing from tests which integer solution is both valid and optimal

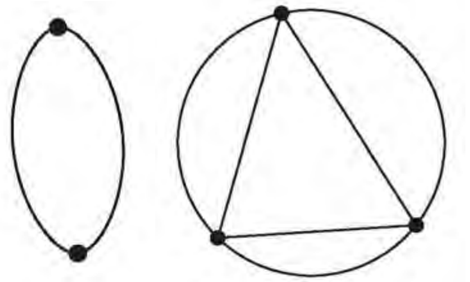
A1ft: Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
3(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
		M1 A1	2.1 1.1b
	The number(s) at the end of activity E indicate this project can be completed in 21 days	A1ft	2.2a
	Critical activities: B, G, I	A1	1.1b
		(4)	
			(7 marks)

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: All boxes completed, number generally increasing L to R (condone one "rogue")
A1: All values cao
A1: Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes)
A1: Critical activities correct

Question	Scheme	Marks	AOs
4(a)	e.g. a graph cannot contain an odd number of odd nodes e.g. number of arcs $=\frac{1+3+4+4+5}{2}=8.5\notin\mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+\left(x+1\right)+\left(2^{x+1}-3\right)+\left(11-x\right)=2\left(18\right)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0\Rightarrow x=...$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0\Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1	2.4
		A1	2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
			(9 marks)

Notes:

(a)

B1: Explanation referring to need for an even number of odd nodes oe

(b)

M1: Forming an equation involving the orders of the 5 odd nodes and $2(18)$

M1: Simplifies to a quadratic in 2^x and attempts to solve

A1: 2 cao

M1: Construct an argument involving the order of the 5 nodes

A1: Explanation considering the number of odd nodes

A1: Deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)

M1: Interprets mathematical language to construct a disconnected graph

A1: Deduce a correct graph

Question	Scheme	Marks	AOs
5	Minimise $(C =)\ 25x + 35y$	B1	3.3
	Subject to: $(500x + 800y \geqslant 150\ 000 \Rightarrow)\ 5x + 8y \geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y) \leqslant x \leqslant \frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b
	(5 marks)		

Notes:

B1: A correct objective function + minimise
B1: Translate information in to a correct inequality
M1: For translating the information given into the LHS inequality
M1: For translating the information given in to the RHS inequality
A1: Simplifying to the correct inequalities

Decision Mathematics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6	$\left(\begin{array}{ccccccc} & P & Q & R & S & T & X \\ A & 32 & 32 & 35 & 34 & 33 & 40 \\ B & 28 & 35 & 31 & 37 & 40 & 40 \\ C & 35 & 29 & 33 & 36 & 35 & 40 \\ D & 36 & 30 & 34 & 33 & 35 & 40 \\ E & 30 & 31 & 29 & 37 & 36 & 40 \\ F & 29 & 28 & 32 & 31 & 34 & 40 \end{array}\right)$	B1	1.1b
	Reducing rows and then columns $\left(\begin{array}{ccccccc} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 2 & 1 & 8 \\ B & 0 & 7 & 3 & 9 & 12 & 12 \\ C & 6 & 0 & 4 & 7 & 6 & 11 \\ D & 6 & 0 & 4 & 3 & 5 & 10 \\ E & 1 & 2 & 0 & 8 & 7 & 11 \\ F & 1 & 0 & 4 & 3 & 6 & 12 \end{array}\right)$ then $\left(\begin{array}{ccccccc} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 3 & 7 & 11 & 4 \\ C & 6 & 0 & 4 & 5 & 5 & 3 \\ D & 6 & 0 & 4 & 1 & 4 & 2 \\ E & 1 & 2 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 4 & 1 & 5 & 4 \end{array}\right)$	M1 A1	1.1b 1.1b
	e.g. augment by 1 then augment by 1 $\left(\begin{array}{ccccccc} & P & Q & R & S & T & X \\ A & 1 & 1 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 2 & 6 & 10 & 3 \\ C & 6 & 0 & 3 & 4 & 4 & 2 \\ D & 6 & 0 & 3 & 0 & 4 & 1 \\ E & 2 & 3 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 3 & 0 & 4 & 3 \end{array}\right)$ followed by $\left(\begin{array}{ccccccc} & P & Q & R & S & T & X \\ A & 2 & 2 & 3 & 1 & 0 & 0 \\ B & 0 & 7 & 1 & 6 & 9 & 2 \\ C & 6 & 0 & 2 & 4 & 3 & 1 \\ D & 6 & 0 & 2 & 0 & 3 & 0 \\ E & 3 & 4 & 0 & 7 & 6 & 3 \\ F & 1 & 0 & 2 & 0 & 3 & 2 \end{array}\right)$	M1 A1ft M1 A1ft A1	1.1b 1.1b 1.1b 1.1b 1.1b
	A – T, B – P, C – Q, (D –), E – R, F – S	A1	2.2a
			(9 marks)

Notes:

B1: cao – introducing a dummy task and appropriate value

M1: Simplifying the initial matrix by reducing rows and then columns

A1: cao

M1: Develop an improved solution – need to see Double covered +e; one uncovered –e ; and one single covered unchanged. 4 lines to 5 lines needed

A1ft: ft on their previous table – no errors

M1: Finding the optimal solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 5 lines needed to 6 lines needed (so getting to the optimal table)

A1ft: ft on their previous table – no errors

A1: cso on final table (so must have scored all previous marks)

A1: cso – this mark is dependent on all M marks being awarded – to deduce the optimal allocation from the location of zeros in the table

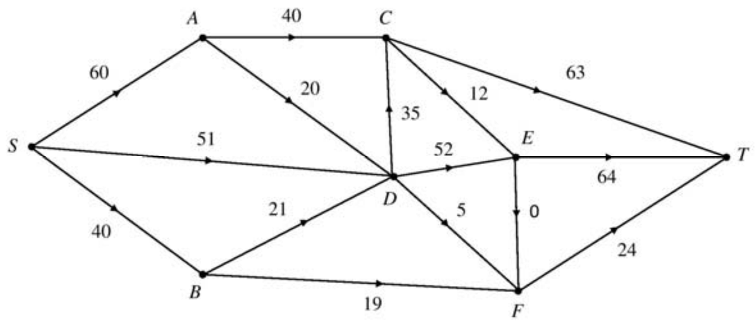
Question	Scheme	Marks	AOs
7(a)	16, 22, 29	B1	1.1b
		(1)	
(b)	$u_{n+1} = u_n + n + 1$	B1	3.3
		(1)	
(c)	As $u_{n+1} = u_n + p(n) \Rightarrow u_n = \lambda n^2 + \mu n + \phi$ and attempt to solve with $n = 1, 2, 3$	M1	1.1b
	$u_n = \frac{1}{2}n(n+1)+1$ 20 101 (regions)	A1 A1ft	1.1b 1.1b
		(3)	
			(5 marks)

Notes:

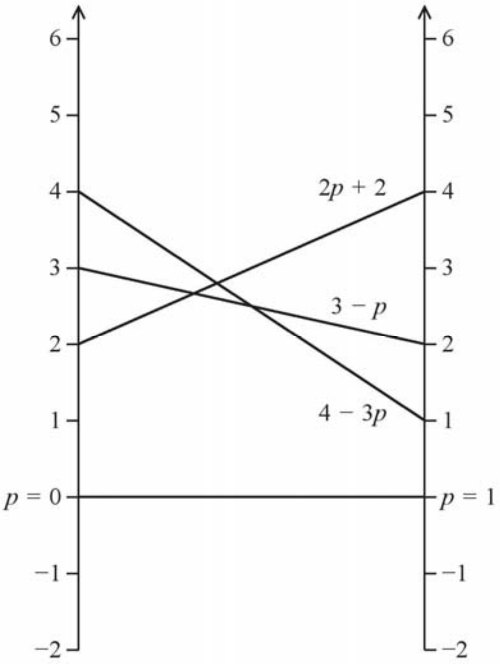
(a)
B1: cao

(b)
B1: Translating problem to mathematical model - correct recurrence relation needed

(c)
M1: An attempt to solve the recurrence relation to determine maximum number of regions
A1: cao
A1ft: Substitution of n = 200 into their quadratic u_n expression

Question	Scheme	Marks	AOs
8(a)	Corridors must be one-way	B1	3.4
		(1)	
(b)	e.g. $55 + x + 40 = 63 + 54 + 24$ **or** $7 + y = 54 + 5$	M1	2.4
	$x = 46$ $y = 52$	A1 A1	1.1b 1.1b
		(3)	
(c)	**(i)** SACET (= 5) SDFET (= 5)	M1 A1	1.1b 1.1b
	(ii) Students must choose SACET, as they cannot travel from F to E	A1	2.2a
		(3)	
(d)		B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem	M1	2.1
	Identification of cut through AC, DC, DE, (EF), FT = 151 value of flow = 151	A1	3.1a
	Therefore it follows that flow is optimal	A1	2.2a
		(3)	
(f)	Consider increasing capacity of arcs in minimum cut	B1	2.1
	Explanation based on a valid argument, such as: • increasing the capacity of any arc other than FT would not increase the flow by more than 1, as total capacity directly in to T is only 152 • increasing the capacity on FT could increase the total flow by 16 (increased flow along SAD, SD and SBD could all be directed through DF to F)	B1	2.4
	Therefore school should choose to widen FT, which could increase the flow through the network by 16	B1	2.2a
		(3)	
		(14 marks)	

Question 8 notes:
(a) **B1:** Explanation of assumption to use this model
(b) **M1:** Either a correct equation, or explanation that flow in = flow out **A1:** cao **A1:** cao
(c) **M1:** One flow augmenting route found from S to T **A1:** Two correct flow augmenting routes 5+ **A1:** Deduce that SACET must be used as students cannot travel from F to E as route is one-way
(d) **B1:** A consistent flow pattern = 151
(e) **M1:** Constructing argument based on max-flow min-cut theorem **A1:** Use appropriate process of finding a minimum cut – cut + value correct **A1:** Correct deduction that the flow is maximal
(f) **B1** Constructing an argument based on arcs in the minimum cut **B1** Detailed explanation as to why choosing anything other than FT does not help **B1** Correct deduction and correct increase in flow of 16

Question	Scheme	Marks	AOs
9(a)	Row minima: 1, 2 max is 2 Column maxima: 4, 4, 3 min is 3	M1 A1	1.1b 1.1b
	Row maximin (2) $\neq$ Column minimax (3) so not stable	A1	2.4
		(3)	
(b)	Let A play strategy 1 with probability *p* and strategy 2 with probability 1- *p*, and using this to get at least one equation in *p*	M1	3.3
	Then if B plays strategy 1, A's gains are $4p + 2(1- p) = 2p + 2$ If B plays strategy 2, A's gains are $p + 4(1- p) = 4 - 3p$ If B plays strategy 3, A's gains are $2p + 3(1- p) = 3 - p$ 	A1 A1	1.1b 1.1b
	Intersection of $2p + 2$ and $3 – p$ occurs where $p = \frac{1}{3}$	dM1 A1ft	1.1b 1.1b
	Therefore player A should play strategy 1 $\frac{1}{3}$ of the time and play strategy 2 $\frac{2}{3}$ of the time	A1ft	3.2a
	The value of the game to player A is $2\frac{2}{3}$	A1	1.1b
		(9)	
			(12 marks)

Question 9 notes:

(a)

M1: Finding row minimums and column maximums – condone one error

A1: Row minima and column maxima correct

A1: Explanation involving $2 \neq 3$ and a conclusion

(b)

M1: Translating situation into model by defining variables and constructing at least one equation

A1: One row correct

A1: All three rows correct

M1: Axes correct, at least one line correctly drawn for their expression

A1: Correct graph

M1: Using their probability expectation graph to find the probability by equating their two correct expressions and attempting to solve as far as $p =$

A1ft: ft on their optimal intersection

A1ft: Interpret their value of p in the context of the question – must refer to play, player A

A1: cao

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