APPLICATIONS OF INTEGRATION ANSWERS EDEXCEL A LEVEL YEAR 1

1.

(a)	Gradient of tangent at P: $m = 4$, Grad. of normal $= -\frac{1}{m}$ $\left(= -\frac{1}{4}\right)$	B1, M1	
	Equation of normal: $y - 4 = -\frac{1}{4}(x-1)$ $(4y = -x + 17)$	M1 A1	(4)
(b)	$(3x-1)^2 = 9x^2 - 6x + 1$	B1	
	Integrate: $\frac{9x^3}{3} - \frac{6x^2}{2} + x$ $(+C)$	M1 A1ft	
	Substitute (1, 4) to find $c = \ldots$, $c = 3$ $(y = 3x^3 - 3x^2 + x + 3)$	M1, A1	(5)
(c)	Gradient of (tangent to) C is ≥ 0	B1	
	Gradient of given line is < 0 (-2)	B1	(2)
			11

2.

$\frac{5x^2+2}{x^{\frac{1}{2}}} = 5x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$	M1: One term correct.	M1 A1	
	A1: Both terms correct, and no extra terms.		
$f(x) = 3x + \frac{5x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \frac{2x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ $(+C)$	(+ C not required here)	M1 A1ft	
$6 = 3 + 2 + 4 + C$	Use of $x = 1$ and $y = 6$ to form eqn. in C	M1	
$C = -3$		A1cso	
$3x + 2x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 3$	(simplified version required)	A1 (ft C)	
			(7)
[or: $3x + 2\sqrt{x^5} + 4\sqrt{x} - 3$ or equiv.]			
		Total 7 marks	

3.

(a) $3x^2 \to cx^3$ or $-6 \to cx$ or $-8x^{-2} \to cx^{-1}$	M1	
$f(x) = \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1}$ $(+C)$ $\left(x^3 - 6x + \frac{8}{x}\right)$	A1 A1	
Substitute $x = 2$ <u>and</u> $y = 1$ into a 'changed function' to form an equation in C.	M1	
$1 = 8 - 12 + 4 + C$ $C = 1$	A1cso	(5)
(b) $3 \times 2^2 - 6 - \frac{8}{2^2}$	M1	
$= 4$	A1	
Eqn. of tangent: $y - 1 = 4(x - 2)$	M1	
$y = 4x - 7$ (Must be in this form)	A1	(4)
		9

4.

(a) $4x \to kx^2$ or $6\sqrt{x} \to kx^{3/2}$ or $\frac{8}{x^2} \to kx^{-1}$ (k a non-zero constant)	M1	
$f(x) = 2x^2,\ -4x^{3/2},\ -8x^{-1}$ $(+ C)$ (+ C not required)	A1, A1, A1	
At $x = 4$, $y = 1$: $1 = (2 \times 16) - \left(4 \times 4^{3/2}\right) - \left(8 \times 4^{-1}\right) + C$ <u>Must be in part (a)</u>	M1	
$C = 3$	A1	(6)
(b) $f'(4) = 16 - (6 \times 2) + \frac{8}{16} = \frac{9}{2}$ $(= m)$ [M: Attempt $f'(4)$ with the <u>given</u> f'. <u>Must be in part (b)</u>]	M1	
Gradient of normal is $-\frac{2}{9}\left(= -\frac{1}{m}\right)$ [M: Attempt perp. grad. rule. Dependent on the use of their $f'(x)$]	M1	
Eqn. of normal: $y - 1 = -\frac{2}{9}(x - 4)$ (or any equiv. form, e.g. $\frac{y-1}{x-4} = -\frac{2}{9}$)	M1 A1	(4)
Typical answers for A1: $\left(y = -\frac{2}{9}x + \frac{17}{9}\right)$ $(2x + 9y - 17 = 0)$ $(y = -0.\dot{2}x + 1.\dot{8})$		
Final answer: gradient $-\frac{1}{(9/2)}$ or $-\frac{1}{4.5}$ is A0 (but all M marks are available).		
		10

5.

$(\mathrm{f}(x)=)\dfrac{3x^3}{3}-\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}-7x(+c)$	M1
$=\ x^3-2x^{\frac{3}{2}}-7x\quad(+c)$	A1A1
$\mathrm{f}(4)=22\ \Rightarrow\ 22=64-16-28+c$	M1
$\boldsymbol{c=2}$	A1cso (5)
	[5]

6.

$x\sqrt{x}=x^{\frac{3}{2}}$ (Seen, or implied by correct integration)	B1
$x^{-\frac{1}{2}}\to kx^{\frac{1}{2}}$ or $x^{\frac{3}{2}}\to kx^{\frac{5}{2}}$ (k a non-zero constant)	M1
$(y=)\ \dfrac{5x^{\frac{1}{2}}}{\frac{1}{2}}\ldots+\dfrac{x^{\frac{5}{2}}}{\frac{5}{2}}\ (+C)$ ("$y=$" and "$+C$" are not required for these marks)	A1... A1
$35=\dfrac{5\times4^{\frac{1}{2}}}{\frac{1}{2}}+\dfrac{4^{\frac{5}{2}}}{\frac{5}{2}}+C$ An equation in C is required (see conditions below). (With their terms simplified or unsimplified).	M1
$C=\dfrac{11}{5}$ or equivalent $2\dfrac{1}{5}$, 2.2	A1
$y=10x^{\frac{1}{2}}+\dfrac{2x^{\frac{5}{2}}}{5}+\dfrac{11}{5}$ (Or equivalent simplified)	A1 ft
I.s.w. if necessary, e.g. $y=10x^{\frac{1}{2}}+\dfrac{2x^{\frac{5}{2}}}{5}+\dfrac{11}{5}=50x^{\frac{1}{2}}+2x^{\frac{5}{2}}+11$	
The final A mark requires an equation "$y=$..." with correct x terms (see below).	**[7]**

7.

$(\mathrm{f}(x)=)\dfrac{12x^3}{3}-\dfrac{8x^2}{2}+x(+c)$	M1 A1 A1
$(\mathrm{f}(-1)=0\Rightarrow)\ 0=4\times(-1)-4\times1-1+c$	M1
$c=\underline{9}$	A1
$\left[\mathrm{f}(x)=4x^3-4x^2+x+9\right]$	
	5

8.

$[\mathrm{f}(x)=]\frac{3x^3}{3}-\frac{3x^2}{2}+5x[+c]$ **or** $\left\{x^3-\frac{3}{2}x^2+5x(+c)\right\}$	M1A1
$10 = 8 - 6 + 10 + c$	M1
$c = -2$	A1
$\mathrm{f}(1) = 1-\frac{3}{2}+5\ \ \text{"}-2\text{"} = \underline{\frac{5}{2}}$ (o.e.)	A1ft (5)
	5 marks

9.

$(\frac{\mathrm{d}y}{\mathrm{d}x}=)\quad -x^3+\text{"}2\text{"}x^{-2}-\text{"}\left(\frac{5}{2}\right)\text{"}x^{-3}$		M1
$(y=)\quad -\frac{1}{4}x^4+\frac{\text{"}2\text{"}x^{-1}}{(-1)}-\text{"}\left(\frac{5}{2}\right)\text{"}\frac{x^{-2}}{(-2)}(+c)$	Raises power correctly on any one term. Any two follow through terms correct.	M1 A1ft
$(y=)\quad -\frac{1}{4}x^4+\frac{2x^{-1}}{(-1)}-\frac{5}{2}\frac{x^{-2}}{(-2)}(+c)$	This is not follow through – must be correct	A1
Given that $y=7$, at $x=1$, then $7=-\frac{1}{4}-2+\frac{5}{4}+c \Rightarrow c=$		M1
So, $(y=)\quad -\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+c,\ c=8$ **or** $(y=)-\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+8$		A1
		[6]
		6 marks