APPLICATIONS OF DIFFERENTIATIONS ANSWERS EDEXCEL A LEVEL YEAR 1

1.

(a)	$\frac{5-x}{x} = \frac{5}{x} - 1 \quad \left(= 5x^{-1} - 1\right)$		M1	
	$\frac{dy}{dx} = 8x - 5x^{-2}$		M1 A1 A1	
	When $x = 1$, $\frac{dy}{dx} = 3$	(*)	A1	(5)
(b)	At P, $y = 8$		B1	
	Equation of tangent: $y - 8 = 3(x - 1)$	$(y = 3x + 5)$ (or equiv.)	M1 A1ft	(3)
(c)	Where $y = 0$, $x = -\frac{5}{3} \quad (= k)$	(or exact equiv.)	M1 A1	(2)
				10

2.

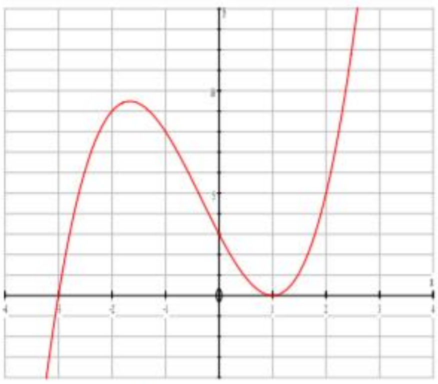
(a) -2 (P), 2 (Q)	(± 2 scores B1 B1)	B1, B1	
			(2)
(b) $y = x^3 - x^2 - 4x + 4$ (May be seen earlier)	Multiply out, giving 4 terms	M1	
$\frac{dy}{dx} = 3x^2 - 2x - 4$	(*)	M1 A1cso	
			(3)
(c) At $x = -1$: $\frac{dy}{dx} = 3(-1)^2 - 2(-1) - 4 = 1$			
Eqn. of tangent: $y - 6 = 1(x - (-1))$, $y = x + 7$	(*)	M1 A1cso	
			(2)
(d) $3x^2 - 2x - 4 = 1$ (Equating to "gradient of tangent")		M1	
$3x^2 - 2x - 5 = 0$ $\quad (3x - 5)(x + 1) = 0$ $\quad x = \ldots$		M1	
$x = \frac{5}{3}$ or equiv.		A1	
$y = \left(\frac{5}{3} - 1\right)\left(\frac{25}{9} - 4\right)$, $\quad = \frac{2}{3} \times \left(-\frac{11}{9}\right) = -\frac{22}{27}$ or equiv.		M1, A1	
			(5)
		Total 12 marks	

3.

(a) $4x \to k$ or $3x^{3/2} \to kx^{1/2}$ or $-2x^2 \to kx$	M1	
$\frac{dy}{dx} = 4 + \frac{9}{2}x^{1/2} - 4x$	A1 A1	(3)
(b) For $x = 4$, $y = (4\times 4) + (3\times 4\sqrt{4}) - (2\times 16) = 16 + 24 - 32 = 8$ (*)	B1	(1)
(c) $\frac{dy}{dx} = 4 + 9 - 16 = -3$ M: Evaluate their $\frac{dy}{dx}$ at $x = 4$	M1	
Gradient of normal = $\frac{1}{3}$	A1ft	
Equation of normal: $y - 8 = \frac{1}{3}(x-4)$, $3y = x + 20$ (*)	M1, A1	(4)
(d) $y = 0$: $x = ...\ (-20)$ and use $(x_2 - x_1)^2 + (y_2 - y_1)^2$	M1	
$PQ = \sqrt{24^2 + 8^2}$ or $PQ^2 = 24^2 + 8^2$ Follow through from $(k, 0)$	A1ft	
May also be scored with $(-24)^2$ and $(-8)^2$.		
$= 8\sqrt{10}$	A1	(3)
		11

4.

(a)



Shape  (drawn anywhere)	B1	
Minimum at (1, 0) (perhaps labelled 1 on x-axis)	B1	
$(-3,0)$ (or -3 shown on $-$ve x-axis)	B1	
(0, 3) (or 3 shown on +ve y-axis)	B1	(4)
N.B. The max. can be anywhere.		
(b) $y = (x+3)(x^2 - 2x + 1)$ [Marks can be awarded if this is seen in part (a)]	M1	
$= x^3 + x^2 - 5x + 3$ $(k = 3)$	A1cso	(2)
(c) $\frac{dy}{dx} = 3x^2 + 2x - 5$	M1 A1	
$3x^2 + 2x - 5 = 3$ or $3x^2 + 2x - 8 = 0$	M1	
$(3x-4)(x+2) = 0$ $x = ...$	M1	
$x = \frac{4}{3}$ (or exact equiv.) , $x = -2$	A1, A1	(6)
		12

5.

$\left(\frac{dy}{dx}=\right) -4+8x^{-2}$ (4 or $8x^{-2}$ for M1… sign can be wrong)	M1A1
$x=2 \Rightarrow \quad m=-4+2=-2$	M1
$y=9-8-\frac{8}{2}=-3$ The first 4 marks <u>could</u> be earned in part (b)	B1
Equation of tangent is: $y+3=-2(x-2) \rightarrow y=1-2x$ (*)	M1 A1cso (6)
Gradient of normal = $\frac{1}{2}$	B1ft
Equation is: $\frac{y+3}{x-2}=\frac{1}{2}$ or better equivalent, e.g. $y=\frac{1}{2}x-4$	M1A1 (3)
$(A:)\ \frac{1}{2}, \quad (B:)\ 8$	B1, B1
Area of triangle is: $\frac{1}{2}(x_B \pm x_A)\times y_P$ with values for all of x_B, x_A and y_P	M1
$\frac{1}{2}\left(8-\frac{1}{2}\right)\times 3=\frac{45}{4}$ or 11.25	A1 (4) **[13]**

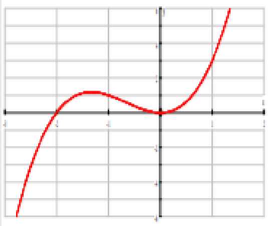
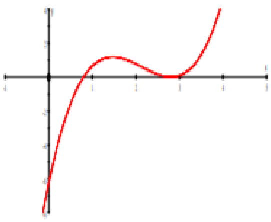
6.

(a) $y=\frac{x^2-5x-24}{x}=x-5-24x^{-1}$ (or equiv., e.g. $x+3-8-\frac{24}{x}$)	M1 A1
$\frac{dy}{dx}=1+24x^{-2}$ or $\frac{dy}{dx}=1+\frac{24}{x^2}$	M1 A1 (4)
(b) $x=2$: $y=-15$ Allow if seen in part (a).	B1
$\left(\frac{dy}{dx}=\right)\ 1+\frac{24}{4}=7$ Follow-through from candidate's <u>non-constant</u> $\frac{dy}{dx}$. This must be simplified to a "single value".	B1ft
$y+15=7(x-2)$ (or equiv., e.g. $y=7x-29$) Allow $\frac{y+15}{x-2}=7$	M1 A1 (4) **[8]**

7.

(a)	$\left(\frac{dy}{dx}=\right)\frac{3}{2}x^2-\frac{27}{2}x^{\frac{1}{2}}-8x^{-2}$	M1A1A1A1 **(4)**
(b)	$x=4 \Rightarrow y=\frac{1}{2}\times 64-9\times 2^3+\frac{8}{4}+30$ $= 32 - 72 + 2 + 30 \qquad = \underline{-8}$ *	M1 A1cso **(2)**
(c)	$x=4 \Rightarrow y'=\frac{3}{2}\times 4^2-\frac{27}{2}\times 2-\frac{8}{16}$ $= 24-27-\frac{1}{2}= \qquad -\frac{7}{2}$ Gradient of the normal $= -1\div "\frac{7}{2}"$ Equation of normal: $y--8=\frac{2}{7}(x-4)$ $\underline{7y-2x+64=0}$	M1 A1 M1 M1A1ft A1 **(6)** **12**

8.

(a)	$[\,y=x^3+2x^2\,]$ so $\frac{dy}{dx}=3x^2+4x$	M1A1 (2)
(b)	 Shape  Touching x-axis at origin Through and not touching or stopping at -2 on x –axis. Ignore extra intersections.	B1 B1 B1 (3)
(c)	At $x=-2$: $\frac{dy}{dx}=3(-2)^2+4(-2)=4$ At $x=0$: $\frac{dy}{dx}=0$ (Both values correct)	M1 A1 (2)
(d)	 Horizontal translation (touches x-axis still) $k-2$ and k marked on positive x-axis $k^2(2-k)$ (o.e) marked on negative y-axis	M1 B1 B1 (3)
		10 marks

9.

11.	$C: y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$	
(a)	So, $y = 2x - 8x^{\frac{1}{2}} + 5$	
	$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}} + \{0\} \qquad (x > 0)$	M1 A1 A1
		[3]
(b)	(When $x = \frac{1}{4}$, $y = 2(\frac{1}{4}) - 8\sqrt{(\frac{1}{4})} + 5$ so) $y = \frac{3}{2}$	B1
	(gradient$= \frac{dy}{dx} =$) $2 - \frac{4}{\sqrt{(\frac{1}{4})}}$ $\{= -6\}$	M1
	Either : $y - "\frac{3}{2}" = "-6"(x - \frac{1}{4})$ **or**: $y = "-6"x + c$ and $"\frac{3}{2}" = "-6"(\frac{1}{4}) + c \Rightarrow c = "3"$	dM1
	So $\underline{y = -6x + 3}$	A1
		[4]
(c)	Tangent at Q is parallel to $2x - 3y + 18 = 0$	
	($y = \frac{2}{3}x + 6 \Rightarrow$) Gradient $= \frac{2}{3}$. so tangent gradient is $\frac{2}{3}$	B1
	So, $"2 - \frac{4}{\sqrt{x}}" = "\frac{2}{3}"$ — Sets their gradient function = their numerical gradient.	M1
	$\Rightarrow \frac{4}{3} = \frac{4}{\sqrt{x}} \Rightarrow x = 9$ — Ignore extra answer $x = -9$	A1
	When $x = 9$, $y = 2(9) - 8\sqrt{9} + 5 = -1$ — Substitutes their found x into equation of curve. $y = -1$.	dM1 A1
		[5]
		12 marks