Mark Scheme (Results)

November 2020

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 02R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where }|pq|=|c| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)\text{ where }|pq|=|c|\text{ and }|mn|=|a| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1 (i)/(ii)	$ar^2 = a + 9d = 48$ or $a + 9d = 4ar$	B1
	$\dfrac{4ar}{ar^2} = 1$	M1
	$\dfrac{4}{r} = 1$	M1
	$r = 4$ $a = 3$ $d = 5$	A1 A1 A1
		[6]
	Alternative method	
	$a_1 + 9d = t_1 r^2$ or $a_1 + 9d = 4t_1 r$	B1
	$r^2 = 4r$	M1
	$r = 4$	A1
	$t_1 r = 12$ So $t_1 = a_1 = 3$	A1
	$a_1 r + 9dr = 48r \Leftrightarrow 12 + 36d = 192 \Leftrightarrow 36d = 180$	M1
	$d = 5$	A1
		[6]

	Notes
B1	For either $ar^2 = a + 9d = 48$ or $a + 9d = 4ar$ oe
M1	For solving simultaneously
M1	For simplifying to $\dfrac{4}{r} = 1$ oe
A1	$r = 4$
A1	$a = 3$
A1	$d = 5$
	Alternative
B1	For either $a_1 + 9d = t_1 r^2$ or $a_1 + 9d = 4t_1 r$ oe
M1	For solving simultaneously
A1	$r = 4$
A1	$a = 3$
M1	For $a_1 r + 9dr = 48r \Leftrightarrow 12 + 36d = 192 \Leftrightarrow 36d = 180$
A1	$d = 5$

Question number	Scheme	Marks
2 (a)	$f(1) = 1 + p + q = -12$	M1
	$p + q = -13$	A1
	$f(4) = 64 + 4p + q = 30$	M1
	$4p + q = -34$	A1
	$3p = -21$	M1
	$p = -7$ and $q = -6$	A1 (6)
(b)	$3^3 - 7\times 3 - 6 = 27 - 21 - 6 = 0$ *	B1 cso (1)
(c)	$(x-3)(x^2+3x+2)$	M1
	$(x-3)(x+2)(x+1)$	M1 A1 (3)
(d)	$x = 3 \quad x = -2 \quad x = -1$	B1 ft (1)
		[11]

	Notes
(a)	
M1	For substitution of 1 into f(x)
A1	For $p + q = -13$ oe
M1	For substitution of 4 into f(x)
A1	For $4p + q = -34$ oe
M1	For solving simultaneously
A1	$p = -7$ and $q = -6$
(b)	
B1cso	For substituting 3 into f(x) and obtaining the given result
(c)	
M1	For $(x-3)(x^2+3x+2)$
M1	For factorising the quadratic
A1	$(x-3)(x+2)(x+1)$
(d)	
B1ft	For $x = 3 \quad x = -2 \quad x = -1$ or follow through part (c)

Question number	Scheme	Marks
3 (a)	$\cos(\angle B)=\dfrac{10^2+8^2-12^2}{2\times10\times8}=\dfrac{10^2+6^2-x^2}{2\times10\times6}$	M1 M1
	$\dfrac{20}{160}=\dfrac{136-x^2}{120}$	
	$x^2=121$	dM1
	$x=11$ *	A1 cso (4)
	Alternative Method	
	$\cos(ADB)=\dfrac{x^2+6^2-10^2}{2\times6\times x}=\dfrac{x^2-64}{12x}$	M1
	$\cos(ADC)=\dfrac{x^2+2^2-12^2}{2\times2\times x}=\dfrac{x^2-140}{4x}$	M1
	$\dfrac{x^2-64}{12x}+\dfrac{x^2-140}{4x}=0$	
	$4x^2=484$	dM1
	$x=11$ *	A1 cso (4)
(b)	Angle ABC / $ABD=\cos^{-1}\left(\dfrac{10^2+8^2-12^2}{2\times10\times8}\right)=82.8^\circ$	B1 M1
	$\dfrac{1}{2}\times10\times6\times\sin 82.8=29.8$	M1 A1 (4) **[8]**
	Alternative Method 1	
	Angle $ADB=\cos^{-1}\left(\dfrac{11^2-64}{12\times11}\right)=64.42^\circ$	B1 M1
	$\dfrac{1}{2}\times6\times11\times\sin 64.42=29.8$	M1 A1 (4)
	Alternative Method 2	
	$\cos ABC=\dfrac{10^2+8^2-12^2}{2\times10\times8}=\dfrac{1}{8}$	B1
	$\sin ABC$ / $ABD=\sqrt{1-\left(\dfrac{1}{8}\right)^2}=\sqrt{\dfrac{63}{64}}=\dfrac{3}{8}\sqrt{7}$	M1
	$\dfrac{1}{2}\times10\times6\times\dfrac{3}{8}\sqrt{7}=29.8$	M1 A1 (4)

	Notes
(a)	
M1	Use of cosine rule to obtain a correct expression for $\cos(\angle B)$. The correct formula in either form may be used.
M1	Use of cosine rule to obtain a second correct expression for $\cos(\angle B)$. The correct formula in either form may be used.
dM1	Dependant on previous M mark - for solving leading to $x^2 = ...$
A1 cso	For obtaining the given result
	Alternative
M1	Use of cosine rule to obtain a correct expression for $\cos(ADB)$. The correct formula in either form may be used.
M1	Use of cosine rule to obtain a correct expression for $\cos(ADC)$. The correct formula in either form may be used.
dM1	Dependant on previous M mark - for solving leading to $4x^2 = ...$
A1	For obtaining the given result
(b)	
B1	For use of the cosine rule to find angle *B*
M1	For $\cos^{-1}\left(\dfrac{10^2+8^2-12^2}{2\times10\times8}\right)$ oe
M1	Use of $\dfrac{1}{2}ab\sin C$ (correct for their angle)
A1	29.8
	Alternative 1
B1	For use of the cosine rule to find angle *ADB*
M1	For $\cos^{-1}\left(\dfrac{11^2-64}{12\times11}\right)$ oe
M1	Use of $\dfrac{1}{2}ab\sin C$ (correct for their angle)
A1	29.8
	Alternative 2
B1	For use of the cosine rule to find cos *B*
M1	Use of $\sin^2 A+\cos^2 A=1$
M1	Use of $\dfrac{1}{2}ab\sin C$ (correct for their angle)
A1	29.8

Question number	Scheme	Marks
4 (a)	x: 0.5, 1, 1.5, 2, 2.5, 3, 3.5; y: 10, 5, 4.89, 5.5, 6.32, 7.22, 8.16	B2 (2)
(b)	Drawn	B1ft B1ft (2)
(c)	$2x+1+\frac{2}{x^2}=8$	M1
	$x=0.6 \qquad x=3.4$	A1 (2)
(d)	$2x+1+\frac{2}{x^2}=\frac{1}{2}x+6$	M1 A1
	$y=\frac{1}{2}x+6$ drawn	M1
	$x=0.7 \qquad x=3.2$	A1 A1 (5)
		[11]

	x	0.5	1	1.5	2	2.5	3	3.5
	y	10	5	4.89	5.5	6.32	7.22	8.16

	Notes
(a)	
B2	For all 4 values correct (B1 for at least 2 values correct)
(b)	
B1ft	For points plotted ft their table (allow half square tolerance)
B1ft	For points joined together with a smooth curve ft their table
(c)	
M1	For $2x+1+\frac{2}{x^2}=8$ may be implied by [y =] 8 identified on the graph
A1	For 0.6 and 3.4 (Allow 0.7 for 0.6)
(d)	
M1	For adding $\frac{1}{2}x$ to both sides of the equation or adding 1 to both sides of the equation
A1	For $2x+1+\frac{2}{x^2}=\frac{1}{2}x+6$
M1	For $y=\frac{1}{2}x+6$ drawn
A1	For 0.7 (allow 0.8)
A1	For 3.2 (allow 3.3)

Question number	Scheme	Marks
5	Triangle OBA: $\tan\left(\frac{\pi}{3}\right)=\frac{AB}{r}$	M1
	$AB = r\tan\left(\frac{\pi}{3}\right)$	A1
	Area of triangle OBA = $\frac{1}{2}r^2\tan\left(\frac{\pi}{3}\right)$	M1
	Area of quadrilateral = $\sqrt{3}r^2$	A1
	Area of sector = $\frac{1}{2}r^2\times\frac{2}{3}\pi=\frac{\pi}{3}r^2$	M1
	Area of shaded region = $\sqrt{3}r^2-\frac{\pi}{3}r^2=10$ oe	dM1
	$r^2=\dfrac{10}{\sqrt{3}-\frac{\pi}{3}}$	M1
	r = 3.82	A1
		[8]

	Notes
Accept angles converted to degrees throughout	
M1	For $\tan\left(\frac{\pi}{3}\right)=\frac{AB}{r}$ or $\tan\left(\frac{\pi}{3}\right)=\frac{AC}{r}$
A1	For $r\tan\left(\frac{\pi}{3}\right)$ (= AB or AC)
M1	For finding the area of either triangle OBA or OCA = $\frac{1}{2}r^2\tan\left(\frac{\pi}{3}\right)$
A1	For finding the area of the quadrilateral = $\sqrt{3}r^2$
M1	For finding the area of the sector = $\frac{1}{2}r^2\times\frac{2}{3}\pi$
dM1	For area of quadrilateral – area of sector = 10 dependant on a reasonable attempt to find both the area of the quadrilateral and the area of the sector
M1	For solving the equation as far as $r^2=\ldots$
A1	For 3.82

Question number	Scheme	Marks
6 (a)	$x^2 - 5x + 4 = (x-4)(x-1)$	M1
	$A(1, 0)$ $B(4, 0)$	A1 A1 (3)
	Notes	
(a) **M1** **A1** **A1**	For solving the quadratic For $A(1, 0)$ For $B(4, 0)$	
(b)	$\frac{dy}{dx} = 2x - 5$	M1
	When $x = 1$ $\frac{dy}{dx} = -3$	A1
	Tangents meet on the axis of symmetry of curve C so $x = \frac{1+4}{2} = \frac{5}{2}$	M1 A1
	When $x = \frac{5}{2}$ $y = -3\left(\frac{5}{2} - 1\right) = -\frac{9}{2}$	M1
	$\left(\frac{5}{2}, -\frac{9}{2}\right)$	A1 (6)
	Notes	
(b)		
M1	For $\frac{dy}{dx} = 2x - 5$ (Allow if seen in part(c))	
A1	$\frac{dy}{dx} = -3$ when $x = 1$	
M1	For use of $\frac{x_1 + x_2}{2}$ or $\frac{-b}{2a}$ or $\frac{dy}{dx} = 0$	
A1	$x = \frac{5}{2}$	
M1	For substitution of $x = \frac{5}{2}$ into $y - y_1 = m(x - x_1)$ oe	
A1	For $\left(\frac{5}{2}, -\frac{9}{2}\right)$	

	Alternative Method $\frac{dy}{dx} = 2x - 5$ When $x = 1$ $\frac{dy}{dx} = -3$ So $y = -3x + 3$ When $x = 4$ $\frac{dy}{dx} = 3$ So $y = 3x - 12$ Meet when $-3x + 3 = 3x - 12$ $\left(\frac{5}{2}, -\frac{9}{2}\right)$	 M1 A1 M1 A1 M1 A1 (6)
	Notes	
 M1 **A1** **M1** **A1** **M1** **A1**	**Alternative** For an attempt to find the equation of the tangent of C at A For $y = -3x + 3$ For an attempt to find the equation of the tangent of C at B For $y = 3x - 12$ For equating the two equations For $\left(\frac{5}{2}, -\frac{9}{2}\right)$	

(c)	Gradient of normal at (0, 1) = $\frac{1}{3}$	M1
	When $x = \frac{5}{2}$ $\quad y = \frac{1}{3}\left(\frac{5}{2} - 1\right) = \frac{1}{2}$	M1
	$\left(\frac{5}{2}, \frac{1}{2}\right)$	A1 (3)
	Alternative Method $y = \frac{1}{3}x - \frac{1}{3}$ and $y = -\frac{1}{3}x + \frac{4}{3}$	M1
	Meet when $\frac{1}{3}x - \frac{1}{3} = -\frac{1}{3}x + \frac{4}{3}$	M1
	$\left(\frac{5}{2}, \frac{1}{2}\right)$	A1 (3)

	Notes
(c)	
M1	For Gradient of normal at (0, 1) = $\frac{1}{3}$ ft the gradient found in part (b)
M1	For substitution of $x = \frac{5}{2}$ into $y - y_1 = m\left(x - x_1\right)$ oe
A1	For $\left(\frac{5}{2}, \frac{1}{2}\right)$
	Alternative
M1	For $y = \frac{1}{3}x - \frac{1}{3}$ and $y = -\frac{1}{3}x + \frac{4}{3}$
M1	For equating the two equations
A1	For $\left(\frac{5}{2}, \frac{1}{2}\right)$

(d)	Area = $\frac{1}{2}(4-1)\times\left(\frac{1}{2}+\frac{9}{2}\right)=\frac{15}{2}$	M1 M1 A1 (3)
	Alternative Method 1 Area = $\frac{1}{2}\times 3\times\frac{1}{2}+\frac{1}{2}\times 3\times\frac{9}{2}=\frac{15}{2}$	M1 M1 A1 (3)
	Alternative Method 2 $AN=\sqrt{1.5^2+0.5^2}=\frac{\sqrt{10}}{2}\quad AT=\sqrt{1.5^2+4.5^2}=\frac{3\sqrt{10}}{2}$	M1
	Area = $2\times\frac{1}{2}\times\frac{\sqrt{10}}{2}\times\frac{3\sqrt{10}}{2}=\frac{30}{4}=\frac{15}{2}$	M1 A1 (3)
	Alternative Method 3 Area = $\frac{1}{2}\begin{vmatrix} 1 & \frac{5}{2} & 4 & \frac{5}{2} & 1 \\ 0 & -\frac{9}{2} & 0 & \frac{1}{2} & 0 \end{vmatrix}\Rightarrow\frac{1}{2}\left(-\frac{9}{2}+2+18-\frac{1}{2}\right)=\frac{15}{2}$	M1 M1 A1 (3) **[15]**

	Notes
(d)	
M1	For $\frac{1}{2}\times AB\times NT$
M1	For $\frac{1}{2}(4-1)\times\left(\frac{1}{2}+\frac{9}{2}\right)$
A1	For $\frac{15}{2}$
	Alternative 1
M1	For area of triangle *ANB* + area of triangle *ATB*
M1	For $\frac{1}{2}\times 3\times\frac{1}{2}+\frac{1}{2}\times 3\times\frac{9}{2}$
A1	$\frac{15}{2}$
	Alternative 2
M1	For finding *AN* and *AT*
M1	For $2\times\frac{1}{2}\times AN\times AT$
A1	$\frac{15}{2}$

	Alternative 3
M1	Use of area = $\frac{1}{2}\begin{vmatrix} 1 & a & 4 & c & 1 \\ 0 & b & 0 & d & 0 \end{vmatrix}$ oe where (a, b) and (c, d) are the coordinates of T and N
M1	$\frac{1}{2}\left(-\frac{9}{2}+2+18-\frac{1}{2}\right)$
A1	$\frac{15}{2}$

Question number	Scheme	Marks
7 (a)	$16-4k(2k-7)\geq 0$	M1
	$2k^2-7k-4\leq 0$	M1
	$(2k+1)(k-4)\leq 0$	M1
	$-\frac{1}{2}\leq k\leq 4$ Accept $-\frac{1}{2}<k<4$	A1 (4)
(b)	$\alpha+\beta=\frac{4}{k}$ $\alpha\beta=\frac{2k-7}{k}$	B1
	$\frac{\alpha+1}{\alpha}+\frac{\beta+1}{\beta}=\frac{2\alpha\beta+\alpha+\beta}{\alpha\beta}=\frac{2\left(\frac{2k-7}{k}\right)+\frac{4}{k}}{\frac{2k-7}{k}}$	M1 M1
	$\frac{4k-10}{k}\times\frac{k}{2k-7}=\frac{2(2k-5)}{2k-7}$	A1
	$\frac{\alpha+1}{\alpha}\times\frac{\beta+1}{\beta}=\frac{\alpha\beta+\alpha+\beta+1}{\alpha\beta}=\frac{\frac{2k-7}{k}+\frac{4}{k}+1}{\frac{2k-7}{k}}$	M1 M1
	$\frac{3k-3}{k}\times\frac{k}{2k-7}=\frac{3(k-1)}{2k-7}$	A1
	$\left(2k-7\right)x^2-2(2k-5)x+3(k-1)=0$	A1 (8)
	ALTERNATIVE METHOD	
	Let $w=\frac{x+1}{x}$	B1
	$x=\frac{1}{w-1}$	M1
	Hence $k\left(\frac{1}{w-1}\right)^2-4\left(\frac{1}{w-1}\right)+2k-7=0$	M1
	$\frac{k}{\left(w-1\right)^2}-\frac{4}{w-1}+2k-7=0$	A1
	$k-4(w-1)+(2k-7)\left(w-1\right)^2=0$	M1
	$k-4w+4+(2k-7)(w^2-2w+1)=0$	M1
	$k-4w+4+2kw^2-4kw+2k-7w^2+14w-7=0$	A1
	$\left(2k-7\right)x^2-2(2k-5)x+3(k-1)=0$	A1 (8) **[12]**

	Notes
(a)	
M1	For use of b^2-4ac (Ignore inequality for this mark)
M1	For a 3 TQ ≤ 0 oe
M1	For solving their 3TQ (Ignore inequality for this mark) May be implied by $-\frac{1}{2}$ and 4 seen as critical values
A1	For $-\frac{1}{2}\leq k\leq 4$ Allow < instead of $\leq$ Accept $-\frac{1}{2}<k<4$
(b)	
B1	For $\alpha+\beta=\frac{4}{k}$ and $\alpha\beta=\frac{2k-7}{k}$
M1	For $\frac{\alpha+1}{\alpha}+\frac{\beta+1}{\beta}=\frac{2\alpha\beta+\alpha+\beta}{\alpha\beta}$
M1	For substitution into $\frac{2\alpha\beta+\alpha+\beta}{\alpha\beta}$ with some attempt to simplify
A1	For $\frac{2(2k-5)}{2k-7}$
M1	For $\frac{\alpha+1}{\alpha}\times\frac{\beta+1}{\beta}=\frac{\alpha\beta+\alpha+\beta+1}{\alpha\beta}$
M1	For substitution into $\frac{\alpha\beta+\alpha+\beta+1}{\alpha\beta}$ with some attempt to simplify
A1	For $\frac{3(k-1)}{2k-7}$
A1	For $\left(2k-7\right)x^2-2(2k-5)x+3(k-1)=0$
	Alternative
B1	For $w=\frac{x+1}{x}$
M1	For rearranging to make *x* the subject
M1	For substitution of $x=\frac{1}{w-1}$ into the quadratic
A1	For $\frac{k}{\left(w-1\right)^2}-\frac{4}{w-1}+2k-7=0$
M1	For multiplying by $\left(w-1\right)^2$
M1	For expanding brackets
A1	For $k-4w+4+2kw^2-4kw+2k-7w^2+14w-7=0$
A1	For $\left(2k-7\right)x^2-2(2k-5)x+3(k-1)=0$

Question number	Scheme	Marks
8	$\log_x 3 = \dfrac{1}{1\text{og}_3 x}$	B1
	Let $y = \log_3 x$	
	So $y - \dfrac{2}{y} = 1$	M1
	$y^2 - y - 2 = 0$	A1
	$(y-2)(y+1) = 0$	M1
	$\log_3 x = 2$ or $\log_3 x = -1$	M1
	$x = 9$ or $x = \dfrac{1}{3}$	A1 A1
		[7]

	Notes
B1	For use of $\log_a x = \dfrac{1}{\log_b a}$
M1	For $y - \dfrac{2}{y} = 1$ oe
A1	For rearranging to a 3 TQ
M1	For solving the 3 TQ
M1	For either $\log_3 x = 2$ or $\log_3 x = -1$
A1	For $x = 9$
A1	For $x = \dfrac{1}{3}$

Question number	Scheme	Marks
9	$u = \mathrm{e}^{-t} \qquad v = \sin 2t$ $u' = -\mathrm{e}^{-t} \qquad v' = 2\cos 2t$ $\frac{\mathrm{d}x}{\mathrm{d}t} = 2\mathrm{e}^{-t}\cos 2t - \mathrm{e}^{-t}\sin 2t = 2\mathrm{e}^{-t}\cos 2t - x$ $u = 2\mathrm{e}^{-t} \qquad v = \cos 2t$ $u' = -2\mathrm{e}^{-t} \qquad v' = -2\sin 2t$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4\mathrm{e}^{-t}\sin 2t - 2\mathrm{e}^{-t}\cos 2t - \frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4x - \left(\frac{\mathrm{d}x}{\mathrm{d}t} + x\right) - \frac{\mathrm{d}x}{\mathrm{d}t} = -5x - 2\frac{\mathrm{d}x}{\mathrm{d}t}$ $\therefore \frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 0 \quad *$	 M1 A1 A1 M1 A1 dM1 dM1 A1 cso **[8]**
	Notes	
M1	For an attempt to differentiate using the product rule. Must have 2 terms added together.	
A1	For one correct term	
A1	For two correct terms	
M1	Attempts to differentiate $\frac{\mathrm{d}x}{\mathrm{d}t}$	
A1	For $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4\mathrm{e}^{-t}\sin 2t - 2\mathrm{e}^{-t}\cos 2t - \frac{\mathrm{d}x}{\mathrm{d}t}$ oe	
dM1	Dependant on previous M mark - for substitution of x and $\frac{\mathrm{d}x}{\mathrm{d}t}$ into $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ or for substitution of $x, \frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ into the given equation	
dM1	Dependant on previous M mark - for simplifying to $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -5x - 2\frac{\mathrm{d}x}{\mathrm{d}t}$ oe or All 5 correct terms seen and an attempt to simplify (5 correct terms may be implied by 7 correct terms)	
A1 cso	Obtains the given equation or clear working to show that the equation = 0	

Question number	Scheme	Marks
10 (a)	$\mathrm{f}(1) = 32(1^3) - 33(1) + 1 = 0$ *	B1 cso (1)
(b)	$(x-1)(32x^2 + 32x - 1) = 0$	M1
	A correct method shown to solve a quadratic e.g. $\dfrac{-32 \pm \sqrt{32^2 + 4\times 32}}{64}$	M1
	(x = 1) or $\dfrac{-4+3\sqrt{2}}{8}$ or $\dfrac{-4-3\sqrt{2}}{8}$ Accept decimals correct to 3 sf e.g. = 0.0303… or = –1.03 …	A1 A1 (4)
(c)	$\sqrt{x} = \dfrac{1}{8x}$	M1
	$p = \dfrac{1}{4}$	A1 (2)

	Notes
(a) **B1 cso**	For substitution of 1 into $\mathrm{f}(x)$ to obtain the given result
(b) **M1**	For $(x-1)(32x^2 + 32x - 1) = 0$
M1	A correct method shown to solve a quadratic. If an algebraic method is not shown then M0A0A0 is awarded.
A1	For $\dfrac{-4+3\sqrt{2}}{8}$ Allow 0.0303 or better
A1	For $\dfrac{-4-3\sqrt{2}}{8}$ Allow -1.03 or better
(c) **M1**	For equating the two equations
A1	For $p = \dfrac{1}{4}$

(d)	$\pi\int_{\frac{1}{4}}^{a} x \text{ d}x = \pi\left[\frac{x^2}{2}\right]_{\frac{1}{4}}^{a}$	M1
	$= \pi\left(\frac{a^2}{2} - \frac{1}{32}\right)$	A1
	$\pi\int_{\frac{1}{4}}^{a} \frac{1}{64x^2} \text{ d}x = \pi\left[-\frac{1}{64x}\right]_{\frac{1}{4}}^{a}$	M1
	$= \pi\left(\frac{1}{16} - \frac{1}{64a}\right)$	A1
	$= \pi\left(\frac{a^2}{2} - \frac{1}{32}\right) - \pi\left(\frac{1}{16} - \frac{1}{64a}\right) = \frac{27\pi}{64}$	dM1
	$32a^3 - 33a + 1 = 0$	A1
	So $a = 1$	A1 (7)
	Alternative Method	
	$\pi\int_{\frac{1}{4}}^{a} \left(x - \frac{1}{64x^2}\right) \text{d}x = \frac{27\pi}{64}$	M1
	$\int_{\frac{1}{4}}^{a} \left(64x - \frac{1}{x^2}\right) \text{d}x = 27$	A1
	$\left[32x^2 + \frac{1}{x}\right]_{\frac{1}{4}}^{a} = 27$	M1 A1
	$\left(32a^2 + \frac{1}{a}\right) - (2+4) = 27$	dM1
	$32a^3 - 33a + 1 = 0$	A1
	So $a = 1$	A1 (7) **[14]**

	Notes
(d)	
M1	For an attempt to integrate $\pi\int_{\frac{1}{4}}^{a} x \, \mathrm{d}x$ Ignore limits
A1	For $\pi\left(\frac{a^2}{2}-\frac{1}{32}\right)$
M1	For an attempt to integrate $\pi\int_{\frac{1}{4}}^{a} \frac{1}{64x^2} \, \mathrm{d}x$ Ignore limits
A1	For $=\pi\left(\frac{1}{16}-\frac{1}{64a}\right)$
M1	Dependant on at least one previous M mark being awarded - for subtraction of the two integrals.
A1	For $32a^3-33a+1=0$ oe
A1	For a = 1
	Alternative
M1	For $\pi\int_{\frac{1}{4}}^{a}\left(x-\frac{1}{64x^2}\right) \mathrm{d}x=\frac{27\pi}{64}$ Ignore limits
A1	For $\int_{\frac{1}{4}}^{a}\left(64x-\frac{1}{x^2}\right)\mathrm{d}x=27$ Ignore limits
M1	For an attempt to integrate $\int_{\frac{1}{4}}^{a}\left(64x-\frac{1}{x^2}\right)\mathrm{d}x$ Ignore limits
A1	$\left[32x^2+\frac{1}{x}\right]_{\frac{1}{4}}^{a}=27$
M1	Dependant on at least one previous M mark being awarded - for correct substitution of the limits
A1	For $32a^3-33a+1=0$ oe
A1	For a = 1