Pearson Edexcel

Mark Scheme (Post-Standardisation V1 – 27/11/20)

Autumn 2020

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \quad \text{leading to } x=\ldots.$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=\ldots.$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0, \quad q\neq 0 \qquad \text{leading to } x=\ldots.$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

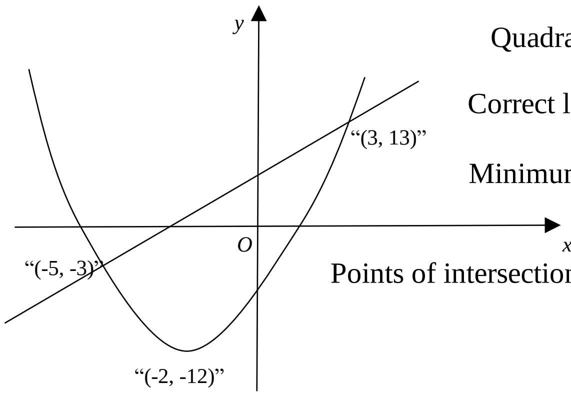
Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

International GCSE Further Pure Mathematics – Paper 1 mark scheme

Question number	Scheme	Marks
1 (a)	$t = \dfrac{10}{3}$ $P = 3 + 2\sin\dfrac{5\pi}{4}$ $= 3 - \sqrt{2}$ oe (e.g. $3 - \dfrac{2}{\sqrt{2}}$)	M1 A1 (2)
(b) (i)	5	B1
(ii)	1	B1 (2)
(c)	$4 = 3 + 2\sin\left(\dfrac{3\pi t}{8}\right)$ $\dfrac{1}{2} = \sin\left(\dfrac{3\pi t}{8}\right)$ $\dfrac{\pi}{6} = \left(\dfrac{3\pi t}{8}\right)$ $t = \dfrac{4}{9}$ oe	M1 M1 A1 (3)
		[7]

Part	Mark	Additional Guidance
(a)	M1	Correct substitution of $t = \dfrac{10}{3}$, leading to an exact value for P, simplification not required.
	A1	Answer stated, oe exact value.
(c)	M1	Correctly substitutes the value of $P = 4$ and rearranges to give an expression of the form $a = \sin\left(\dfrac{3\pi t}{8}\right)$
	M1	Correctly uses the inverse sin function to arrive at $b = \left(\dfrac{3\pi t}{8}\right)$ and solves to find a value of $\dfrac{3\pi t}{8}$, allow this value to be in degrees **Do not allow this mark if *b* > 1 or *b* < -1**
	A1	oe

Question number	Scheme	Marks
2 (a)	$(x+2)^2 - 4 - 8$	M1
	$(x+2)^2 - 12 \qquad a = 2 \quad b = -12$	A1 (2)
(b)	$x^2 + 4x - 8 = 2x + 7$	M1
	$x^2 + 2x - 15 = 0$	dM1
	$(x-3)(x+5) = 0$ or any valid method	M1
	$x = 3$, $y = 13$ $\qquad\qquad$ $x = -5$, $y = -3$	A1 A1 (5)
(c)	 Quadratic drawn	B1
	Correct line drawn	B1
	Minimum labelled	B1 ft
	Points of intersection labelled	B1 ft (4)
		[11]

Part	Mark	Additional Guidance
(a)	M1	Use general guidance, allow an expression of the form $\left(x \pm \frac{4}{2}\right)^2 \pm q \pm 8 \qquad q \neq 0$
	A1	Correct expression as shown, *a* and *b* need not be explicitly stated
(b)	M1	Correctly equates the 2 expressions
	dM1	Rearranges to a 3TQ = 0 (allow any 3TQ if intention of rearrangement is clear)
	M1	Uses any valid method to solve – **see general guidance**
	A1	For either pair of values stated
	A1	For all four values, correctly paired or written as coordinates.
(c)	B1	Correctly shaped quadratic curve, with a clear minimum point, drawn anywhere on their axis, mark intention.
	B1	Correct line – **must have a positive y intercept, a positive gradient and a negative x intercept**
	B1ft	Correctly labelled coordinates for their minimum, ft their answer from b, **must correctly ft their answer from a, ie minimum point labelled (- *a*, *b*)**
	B1ft	Correctly labelled coordinates for their intersections.

Question number	Scheme	Marks
3	$\ln 12 = \ln a + (2-1)\ln b$ oe	M1
	$ab = 12$ oe	A1
	$\ln 768 = \ln a + (5-1)\ln b$ oe	M1
	$ab^4 = 768$ oe	A1
	$\dfrac{768}{12} = \dfrac{ab^4}{ab}$ $(b^3 = 64)$	ddM1
	$b = 4$ $a = 3$	A1 A1
	ALTERNATIVE METHOD	
	$\ln a + (2-1)\ln b = \ln 12$ oe	M1 A1
	$\ln a + (5-1)\ln b = \ln 768$ oe	
	$3\ln b = \ln b^3$ $\ln 768 - \ln 12 = \ln 64$	M1 A1
	$b^3 = 64$	M1
	$b = 4$	A1
	$a = 3$	A1
		[7]

Part	Mark	Additional Guidance
	M1	Correct equation as shown oe
	A1	Correct equation as shown oe
	M1	Correct equation as shown oe
	A1	Correct equation as shown oe
	ddM1	**Dependent on both previous method marks**, uses any clear, valid method to reduce to an equation in *a* (or less likely, *b*)
	A1	For correct *b*
	A1	For correct *a*
ALT	M1	One correct equation as shown oe
	A1	Both correct equations as shown oe
	M1	Clear attempt to subtract one equation from the other
	A1	Achieves the two terms shown
	ddM1	**Dependent on both previous method marks,** eliminates the logs and achieves an equation in *b* only
	A1	For correct *b*
	A1	For correct *a*
	Allow full marks in general for just $b = 4$ and $a = 3$	

Question number	Scheme	Marks
4 (a)	$\dfrac{dy}{dx} = 3x^2 - 6x - 24$	M1
	$"(x+2)(x-4)" = 0$	M1
	$x = -2$ or $x = 4$	A1
	(–2, 34) and (4, –74)	A1 (4)
(b)	$\dfrac{d^2y}{dx^2} = "6x-6"$ and substitution of –2 or 4 or consider values of $\dfrac{dy}{dx}$ either side of –2 or 4 or use properties of a cubic*	M1
	(–2, 34) maximum and (4, –74) minimum	A1cso (2) **[6]**

Part	Mark	Additional Guidance
(a)	M1	General guidance – an attempt to differentiate, power of at least one term must decrease by one. Also, no power must increase.
	M1	Equates their derivative = 0 and attempts to solve by any method **– see general guidance.**
	A1	Both correct values for *x*
	A1	All values correct, listed as coordinates or correctly paired *x* =….. *y* =.....
(b)	M1	**Correctly differentiates their derivative** from part a
	A1cso	Must correctly make the argument they've chosen and identify the points as shown. If substituting *x* = -2 and *x* = 4 or values either side of the these, their evaluations of the substitutions must be correct. * A convincing argument about the shape of a positive cubic curve and position of the minimum and maximum point must be made. Send to review, if in doubt.

Question number	Scheme	Marks
5 (a)	$1+\frac{1}{2}(-x)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(-x)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-x)^3$	M1
	$1-\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^3$	A1 A1 (3)
(b)	$x = 0.08$	B1
	$1-\frac{1}{2}\text{"}(0.08)\text{"}-\frac{1}{8}\text{"}(0.08)\text{"}^2-\frac{1}{16}\text{"}(0.08)\text{"}^3$	M1
	0.959168 cao	A1 cao (3)
(c)	$(\sqrt{0.92}=\frac{\sqrt{23}}{5}\Rightarrow\sqrt{23}=)\text{"}0.959168\text{"}\times 5$	M1
	4.79584	A1 (2)
		[8]

Part	Mark	Additional Guidance
(a)	M1	For an attempt at a Binomial expansion. An attempt is defined as the following • The expansion must start with 1 • The powers of their $-x$ must be correct • $-x$ must be used at least once • The denominators 2! And 3! must be seen. Accept 2 and 6 Can be implied by at least 2 correct terms in an expansion
	A1	For at least one term in x correct and fully simplified.
	A1	For the expansion fully correct and simplified.
(b)	B1	For finding the value of $x = 0.08$
	M1	For correctly substituting **their** value of x into the expansion **provided** $\|x\| < 1$ Use of **their expansion** or the correct expansion must be seen explicitly here
	A1	cao
(c)	M1	For use of their value from (b) in $\sqrt{0.92}=\frac{\sqrt{23}}{5}\Rightarrow\sqrt{23}=\text{"}0.959168\text{"}\times 5$
	A1	cao

Question number	Scheme	Marks
6 (a)	$(\sin A\cos B+\cos A\sin B)+(\sin A\cos B-\cos A\sin B)$ $= 2\sin A\cos B$ *	M1 A1 cso (2)
(b)	$\sin 8x+\sin 6x$	B1 (1)
(c)	$3\int_0^{\frac{\pi}{4}}(\sin 8x+\sin 6x)\,\mathrm{d}x$	M1
	$= (3)\left[-\frac{1}{8}\cos 8x-\frac{1}{6}\cos 6x\right]_0^{\frac{\pi}{4}}$	A1
	$= (3)\left[\left(-\frac{1}{8}-0\right)-\left(-\frac{1}{8}-\frac{1}{6}\right)\right]$	M1
	$= \frac{1}{2}$ cao oe	A1cao (4)
		[7]

Part	Mark	Additional Guidance
(a)	M1	Correct expression show.
	A1	cso
(b)	B1	For the expression shown
(c)	M1	$k\int_0^{\frac{\pi}{4}}(\sin 8x+\sin 6x)\,\mathrm{d}x$ $k\neq 0\,\text{or}\,1$ k must be an integer
	A1	Correctly integrated, the 3 need not be present for this mark
	M1	Correctly shown substitution of limits, the 3 need not be present for this mark, need not be simplified. *This mark can be implied if first M1 A1 awarded and final correct answer*
	A1	cao oe

Question number	Scheme	Marks
7 (a)	$(V=)x^3 \quad (\frac{\mathrm{d}V}{\mathrm{d}x}=)3x^2 \quad$ (at $x = 2$ $\quad \frac{\mathrm{d}V}{\mathrm{d}x}=12$)	M1
	$\frac{\mathrm{d}x}{\mathrm{d}t}=0.1$	B1 (A1 on ePen)
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}="12"\times 0.1$	M1
	1.2 m^3/s cao oe	A1cao (4)
(b)	(Surface Area =) $6x^2 \quad (\frac{\mathrm{d}A}{\mathrm{d}x})=12x$ at $x = 6$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}x}=72$	A1
	$\frac{\mathrm{d}V}{\mathrm{d}x}=108 \quad \frac{\mathrm{d}A}{\mathrm{d}t}=0.05$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t}="108"\times"\frac{1}{72}"\times 0.05$	M1
	0.075 m^3/s	A1 (5) **[9]**

Part	Mark	Additional Guidance
(a)	M1	Correct expression for Volume, attempt at differentiation to ax^2, a is an integer, $a > 1$.
	B1	$\frac{\mathrm{d}x}{\mathrm{d}t}=0.1$, can be explicit or implicitly used in a chain rule.
	dM1	For **any correct chain rule**, that would lead to a value for $\frac{\mathrm{d}V}{\mathrm{d}t}$ and substitution of 0.1 and their value for $\frac{\mathrm{d}V}{\mathrm{d}x}$
	A1	cao oe
(b)	M1	Correct expression for Surface Area, attempt at differentiation to bx, b is an integer, $b > 1$.
	A1	$\frac{\mathrm{d}A}{\mathrm{d}x}=72$
	B1	$\frac{\mathrm{d}V}{\mathrm{d}x}=108$ and $\frac{\mathrm{d}A}{\mathrm{d}t}=0.05$ clearly stated, implicitly in a chain rule or explicitly
	dM1	For **any correct chain rule**, that would lead to a value for $\frac{\mathrm{d}V}{\mathrm{d}t}$ and substitution of 0.05 and their values for $\frac{\mathrm{d}A}{\mathrm{d}x}$ and $\frac{\mathrm{d}V}{\mathrm{d}x}$
	A1	cao

Question number	Scheme	Marks
8 (a)	$\alpha+\beta=\frac{1}{3} \qquad \alpha\beta=\frac{4}{3}$	B1
	$p=\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$	M1
	$=\frac{1}{3}+\frac{\frac{1}{3}}{\frac{4}{3}}=\frac{7}{12}$ *	A1 cso* (3)
(b)	$q=\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)=\alpha\beta+\frac{1}{\alpha\beta}+\frac{\beta^2+\alpha^2}{\alpha\beta}$	M1
	$=\alpha\beta+\frac{1}{\alpha\beta}+\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	A1 (M1 on ePen)
	$=\frac{4}{3}+\frac{3}{4}+\frac{\frac{1}{9}-\frac{8}{3}}{\frac{4}{3}}=\frac{1}{6}$ oe	dM1 A1 (4) **[7]**

Part	Mark	Additional Guidance
(a)	B1	$\alpha+\beta=\frac{1}{3}$, $\alpha\beta=\frac{4}{3}$ can be explicit or implicitly used later
	M1	For an initial expression correctly adding the roots of g(x) and an attempt to simplify. **Minimum attempt must involve terms $\alpha+\beta$ and an attempt to simplify, bringing the fraction part to a common denominator $\alpha\beta$ written as a single fraction.**
	A1*	cso*
(b)	M1	For multiplying the roots of g(x). **Minimum attempt must involve a correct multiplication of the brackets with terms $\alpha\beta+\frac{1}{\alpha\beta}$ and bringing the fraction part to a common denominator $\alpha\beta$**
	A1 (M1 ePen)	Correct expression as shown.
	dM1	Substitution of their values for $\alpha+\beta$ and $\alpha\beta$ into their expression (this expression must be ready for substitution of $\alpha+\beta$ and $\alpha\beta$)
	A1	$\frac{1}{6}$ oe

Question number	Scheme	Marks
9	$\dfrac{2^{4x}}{2^{3y}} = \dfrac{1}{2^2}$	M1
	$2^{4x-2y} = 2^{-2}$ $\quad (\to\ 4x-3y=-2\ )$	dM1
	$2^{2x}2^{y} = 2^{4}$	M1
	$2^{2x+y} = 2^{4}$ $\quad \to\ (2x+y=4)$	dM1
	A fully correct method using for solving simultaneously leading to either $10x = 10$ **or** $5y = 10$ $4x-3y=-2 \Rightarrow 10x=10$ or $4x-3y=-2 \Rightarrow 5y=10$ $6x+3y=12$ $\qquad$ $4x+2y=8$	ddddM1
	$y=2$ $x=1$	A1 A1 **[7]**
	Alternative Method $4^{x} = \dfrac{16}{2^{y}}$	M1
	$\dfrac{4^{2x}}{8^{y}} = \dfrac{1}{4}$	M1
	$\left(\dfrac{16}{2^{y}}\right)^{2} \times \dfrac{1}{8^{y}} = \dfrac{1}{4}$	ddM1
	$8^{y} \times 2^{2y} = 4 \times 16^{2}$	dddM1
	$2^{3y} \times 2^{2y} = 2^{2} \times 2^{8}$	ddddM1
	$(\ 2^{5y} = 2^{10}\ )$ $\quad y=2$	A1
	$(\ 4^{x} \times 4 = 16\ )$ $\quad x=1$	A1

Part	Mark	Additional Guidance
(a)	M1	For correctly changing any two indices into powers of 2 and simplifying. Accept any two of 2^2 or 2^{4x} or 2^{3y}
	dM1	**Dependent on previous method mark.** A fully correct method using index laws to simplify their expressions as powers of 2 and an attempt to write this as a linear equation.
	M1	For correctly changing both indices to powers of 2, as shown
	dM1	**Dependent on previous method mark.** A fully correct method using index laws to simplify their expressions as powers of 2 and an attempt to write this as a linear equation.
	ddddM1	**Dependent on all previous method marks**
	A1	$y = 2$
	A1	$x = 1$
ALT	M1	For a correct rearrangement of the 2^{nd} equation as shown
	M1	For converting the 16^x into 4^{2x} as shown.
	ddM1	**Dependent on both previous method marks.** Substitution of $\frac{16}{2^y}$ into the second equation, this need not be fully simplified.
	dddM1	**Dependent on all previous method marks.** An attempt to rearrange the equation, must have at least one side of the equation shown correct.
	ddddM1	**Dependent on all previous method marks.** An attempt to convert **all** into powers of 2, must see at least 2 of $2^{3y}, 2^{2y}, 2^2, 2^8$ correctly written.
	A1	$y = 2$
	A1	$x = 1$

Question number	Scheme	Marks
10 (a)	$(x+\frac{\pi}{3})=\frac{\pi}{3}$ or $\frac{2\pi}{3}$ or $\frac{7\pi}{3}$	M1
	$(x+\frac{\pi}{3})=\frac{\pi}{3}$ and $\frac{2\pi}{3}$ and $\frac{7\pi}{3}$	A1
	$x=0,\frac{\pi}{3},2\pi$	A1 (3)
(b)	$\tan\theta=-\frac{5}{3}$	M1
	θ = –59°, –239°, 121°, 301°	M1 A1 (3)
(c)	$1+\sin 2y-2(1-\sin^2 2y)=0$	M1
	$2\sin^2 2y+\sin 2y-1=0$	A1
	$(\sin 2y+1)(2\sin 2y-1)=0$	
	$\sin 2y=-1$ or $\sin 2y=\frac{1}{2}$	dM1
	$2y$ = –90°, (30°), (150°), –330°, –210°	A1
	y = –45°, – 105°, –165°	A1 (5)
		[11]

Part	Mark	Additional Guidance
(a)	M1	For any one of the three indicated angles, ignore any other angles.
	A1	For all three indicated angles, ignore other angles.
	A1	For all three angles, ignore angles out of range, A0 if additional angles in range.
(b)	M1	For $\tan\theta = k$. $k\neq 0$, $k\neq\pm 1$
	M1	Any one correct value, does not need to be to the nearest degree.
	A1	For all four angles, ignore angles out of range, A0 if additional angles in range. All four angles must be given to the nearest degree.
(c)	M1	For the correct use of $1-\sin^2 2y$ in the equation on the left or right side, equation doesn't need to be = 0.
	A1	Correct 3TQ, must be = 0 and a valid attempt to solve leading to sin 2y =
	dM1	$\sin 2y=-1$ or $\sin 2y=\frac{1}{2}$
	A1	For any 3 of the values shown (including the ones in brackets)
	A1	For all 3 values shown. **Rounding answers (where accuracy is specified in the question)** Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
11 (a)	$\mathbf{b}-\mathbf{a}$	B1 (1)
(b)	$\overrightarrow{OZ}=\overrightarrow{OB}+\lambda\,\overrightarrow{BX}\,(=\mathbf{b}+\lambda(-\mathbf{b}+2\mathbf{a}))$	M1
	$=(1-\lambda)\mathbf{b}+2\lambda\mathbf{a}$	A1
	$\overrightarrow{OZ}=\overrightarrow{OA}+\mu\,\overrightarrow{AY}\,(=\mathbf{a}+\mu(-\mathbf{a}+3\mathbf{b}))$	M1
	$=(1-\mu)\mathbf{a}+3\mu\mathbf{b}$	A1
	$(1-\lambda)\mathbf{b}+2\lambda\mathbf{a}=(1-\mu)\mathbf{a}+3\mu\mathbf{b}$	ddM1
	$2\lambda=1-\mu$ $3\mu=1-\lambda$	A1
	$3(1-2\lambda)=1-\lambda$ or $2(1-3\mu)=1-\mu$	M1
	$\lambda=\frac{2}{5}$ or $\mu=\frac{1}{5}$	A1
	$\overrightarrow{OZ}=\frac{1}{5}(4\mathbf{a}+3\mathbf{b})$	A1 (9)
(c)	$\overrightarrow{OM}=p\text{"}\frac{1}{5}(4\mathbf{a}+3\mathbf{b})\text{"}$ and $\overrightarrow{OM}=2\mathbf{a}+q(-2\mathbf{a}+3\mathbf{b})$	M1
	$\frac{4p}{5}=2-2q$ and $\frac{3p}{5}=3q$	M1
	(Solving these equations leads to $p=\frac{5}{3}$)	
	$\overrightarrow{OM}=\frac{1}{3}(4\mathbf{a}+3\mathbf{b})$	A1 (3)
		[13]

Part	Mark	Additional Guidance
(a)	B1	For the indicated vector
(b)	M1	For any correctly written vector path
	A1	For the vector shown
	M1	For any correctly written vector path
	A1	For the vector shown
	ddM1	Equates their 2 vectors – this mark may be implicit in the candidate equating the two components of their 2 vectors, **dependent on the first two method marks.**
	A1	Correct equations as shown
	ddM1	Full and correct method to solve their two simultaneous equations, either by substitution as shown or by elimination. There must be no errors in the method to eliminate γ or μ, **dependent on the first two method marks.**
	A1	Correct value for γ or μ
	A1	Correct vector.
(c)	M1	For the two correct vectors shown, allow use of their $\overrightarrow{OZ}$
	dM1	Correctly equating the components of their vectors for $\overrightarrow{OZ}$ and arriving at a value for p or q
	A1	For the correct vector, as shown.

Question number	Scheme	Marks
12 (a)	$(2\cos x = 0)$ $\quad (x =)\dfrac{\pi}{2}$	B1 (1)
(b)	$(2\cos x = 2\sin x)$ $\quad \tan x = 1$	M1
	$x = \dfrac{\pi}{4}$	A1 (2)
(c)	$\int_{(0)}^{(\text{"}\frac{\pi}{4}\text{"})}(2\sin x)\mathrm{d}x + \int_{(\text{"}\frac{\pi}{4}\text{"})}^{(\text{"}\frac{\pi}{2}\text{"})}(2\cos x)\mathrm{d}x$	M1
	$[-2\cos x]_0^{\frac{\pi}{4}} + [2\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	A1
	$\left(-\sqrt{2}+2\right)+\left(2-\sqrt{2}\right)=4-2\sqrt{2}$	dM1
	$= 4-\sqrt{8}$ cao	A1cao cso (4) **[7]**

Part	Mark	Additional Guidance
(a)	B1	For $\dfrac{\pi}{2}$ or 90 degrees
(b)	M1	For $\tan x = 1$
	A1	For $\dfrac{\pi}{4}$ or 45 degrees
(c)	M1	For both integrals correctly shown, with an addition sign between them. Limits need not be shown.
	A1	For both functions correctly integrated. Limits need not be shown.
	dM1	For their limits clearly and correctly substituted in or for the numerical expression(s) shown in the MS. If mark awarded for substitution, both integrated expressions must have both limits correctly substituted. 0 must be the lower limit on the first integral. Allow ft of their $\dfrac{\pi}{4}$ (must be the upper limit on the first integral and the lower limit on the second and can be in degrees) and their $\dfrac{\pi}{2}$ (must be the upper limit on the second integral and can be in degrees).
	A1	cao cso A0 if degrees used in part c