Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

AS
FURTHER MATHEMATICS

Paper 1

Monday 14 May 2018 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Answer **all** questions in the spaces provided.

1 $z = 3 - \mathrm{i}$

Determine the value of zz^*

Circle your answer.

[1 mark]

10 $\quad\quad$ $\sqrt{10}$ $\quad\quad$ $10 - 2\mathrm{i}$ $\quad\quad$ $10 + 2\mathrm{i}$

2 Three matrices **A**, **B** and **C** are given by

$\mathbf{A} = \begin{bmatrix} 5 & 2 & -3 \\ 0 & 7 & 6 \\ 4 & 1 & 0 \end{bmatrix}$, $\quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 3 & -5 \\ -2 & 6 \end{bmatrix}$ $\quad$ and $\mathbf{C} = \begin{bmatrix} 6 & 4 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

Which of the following **cannot** be calculated?

Circle your answer.

[1 mark]

AB $\quad\quad$ **AC** $\quad\quad$ **BC** $\quad\quad$ $\mathbf{A}^2$

3 Which of the following functions has the fourth term $-\frac{1}{720}x^6$ in its Maclaurin series expansion?

Circle your answer.

[1 mark]

$\sin x$ $\quad\quad$ $\cos x$ $\quad\quad$ e^x $\quad\quad$ $\ln(1 + x)$

4 Sketch the graph given by the polar equation

$$r = \frac{a}{\cos\theta}$$

where a is a positive constant.

[2 marks]

5 Describe fully the transformation given by the matrix $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[3 marks]

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6 (a) Matthew is finding a formula for the inverse function $\operatorname{arsinh} x$.
He writes his steps as follows:

$$\text{Let } y = \sinh x$$

$$y = \frac{1}{2}(e^{x} - e^{-x})$$

$$2y = e^{x} - e^{-x}$$

$$0 = e^{x} - 2y - e^{-x}$$

$$0 = (e^{x})^{2} - 2ye^{x} - 1$$

$$0 = (e^{x} - y)^{2} - y^{2} - 1$$

$$y^{2} + 1 = (e^{x} - y)^{2}$$

$$\pm\sqrt{y^{2} + 1} = e^{x} - y$$

$$y \pm \sqrt{y^{2} + 1} = e^{x}$$

To find the inverse function, swap x and y: $x \pm \sqrt{x^{2} + 1} = e^{y}$

$$\ln\left(x \pm \sqrt{x^{2} + 1}\right) = y$$

$$\operatorname{arsinh} x = \ln\left(x \pm \sqrt{x^{2} + 1}\right)$$

Identify, and explain, the error in Matthew's proof.

[2 marks]

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6 (b) Solve $\ln\left(x+\sqrt{x^2+1}\right)=3$

[1 mark]

7 Find two invariant points under the transformation given by $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

[2 marks]

Turn over ▶

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8 $2 - 3\mathrm{i}$ is one root of the equation

$$z^3 + mz + 52 = 0$$

where m is real.

8 (a) Find the other roots.

[3 marks]

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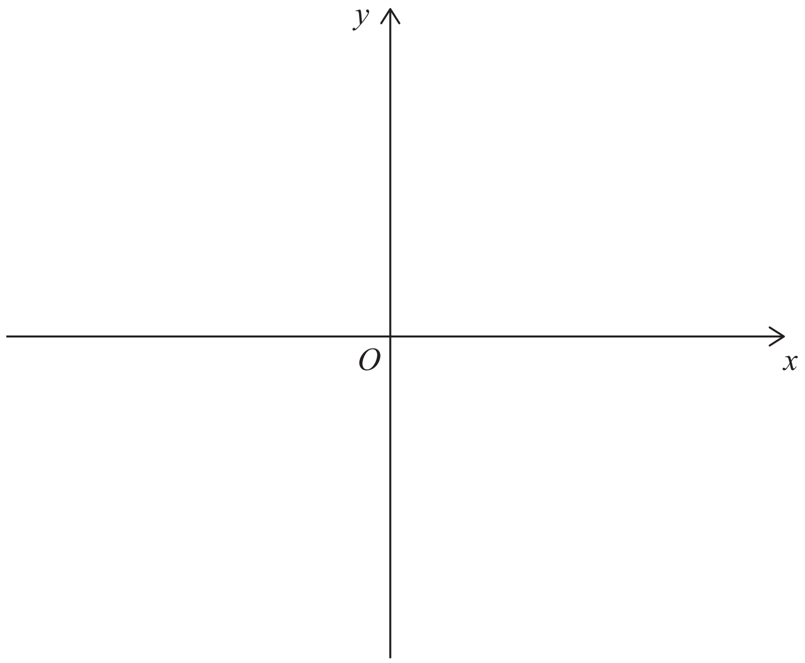
8 (b) Determine the value of m.

[2 marks]

Turn over ▶

9 (a) Sketch the graph of $y^2 = 4x$

[1 mark]



9 (b) Ben is using a 3D printer to make a plastic bowl which holds exactly 1000 cm^3 of water.

Ben models the bowl as a region which is rotated through 2π radians about the x-axis.

He uses the finite region enclosed by the lines $x = d$ and $y = 0$ and the curve with equation $y^2 = 4x$ for $y \geq 0$

9 (b) (i) Find the depth of the bowl to the nearest millimetre.

[4 marks]

9 (b) (ii) What assumption has Ben made about the bowl?

[1 mark]

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10 (a) Prove by induction that, for all integers $n \geq 1$,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

[4 marks]

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10 (b) **Hence** show that

$$\sum_{r=1}^{2n} r(r-1)(r+1) = n(n+1)(2n-1)(2n+1)$$

[4 marks]

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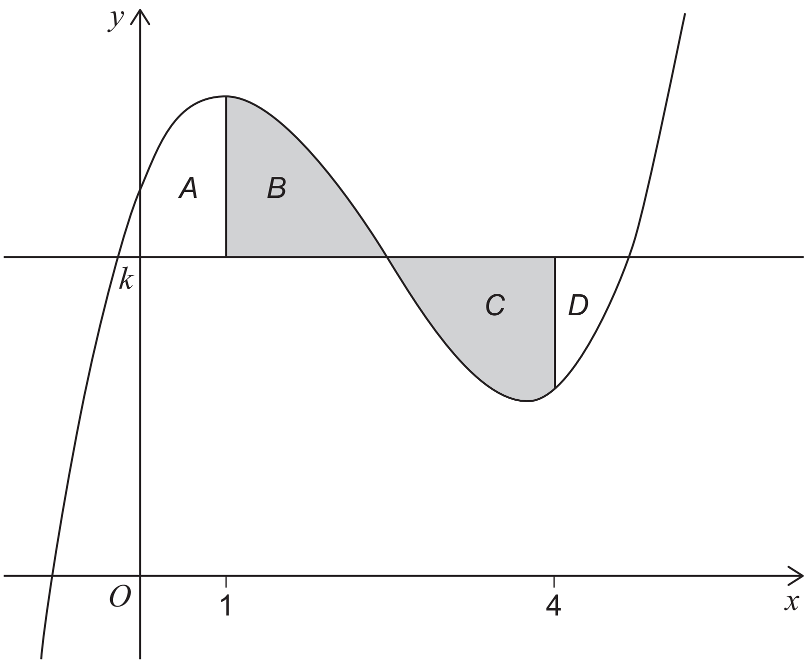
11 Four finite regions A, B, C and D are enclosed by the curve with equation

$$y = x^3 - 7x^2 + 11x + 6$$

and the lines $y = k$, $x = 1$ and $x = 4$, as shown in the diagram below.



The areas of B and C are equal.

Find the value of k.

[3 marks]

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12 (a) Show that the matrix $\begin{bmatrix} 5-k & 2 \\ k^3+1 & k \end{bmatrix}$ is singular when $k = 1$.

[1 mark]

12 (b) Find the values of k for which the matrix $\begin{bmatrix} 5-k & 2 \\ k^3+1 & k \end{bmatrix}$ has a negative determinant.

Fully justify your answer.

[5 marks]

13 The graph of the rational function $y = \mathrm{f}(x)$ intersects the x-axis exactly once at $(-3, 0)$

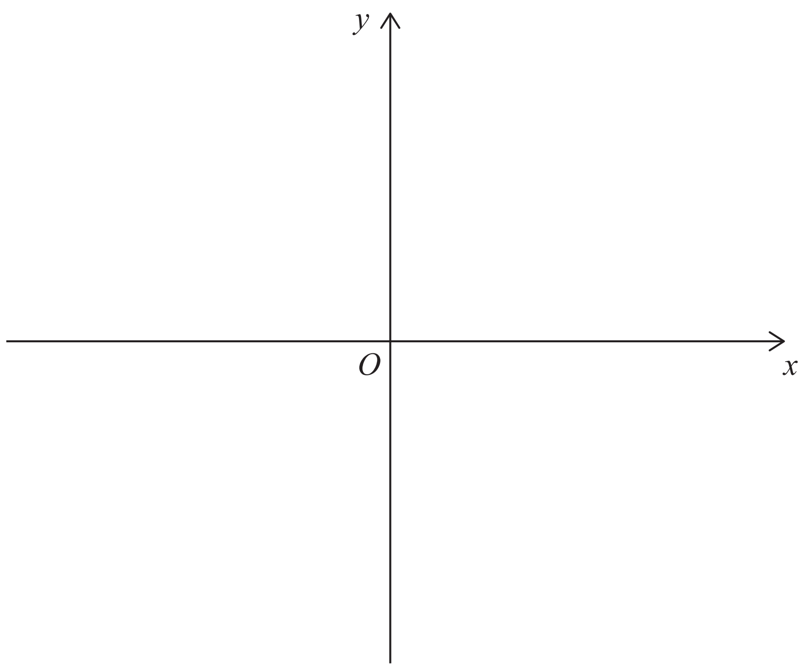
The graph has exactly two asymptotes, $y = 2$ and $x = -1$

13 (a) Find $\mathrm{f}(x)$

[2 marks]

13 (b) Sketch the graph of the function.

[3 marks]



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13 (c) Find the range of values of x for which $\mathrm{f}(x) \leq 5$

[4 marks]

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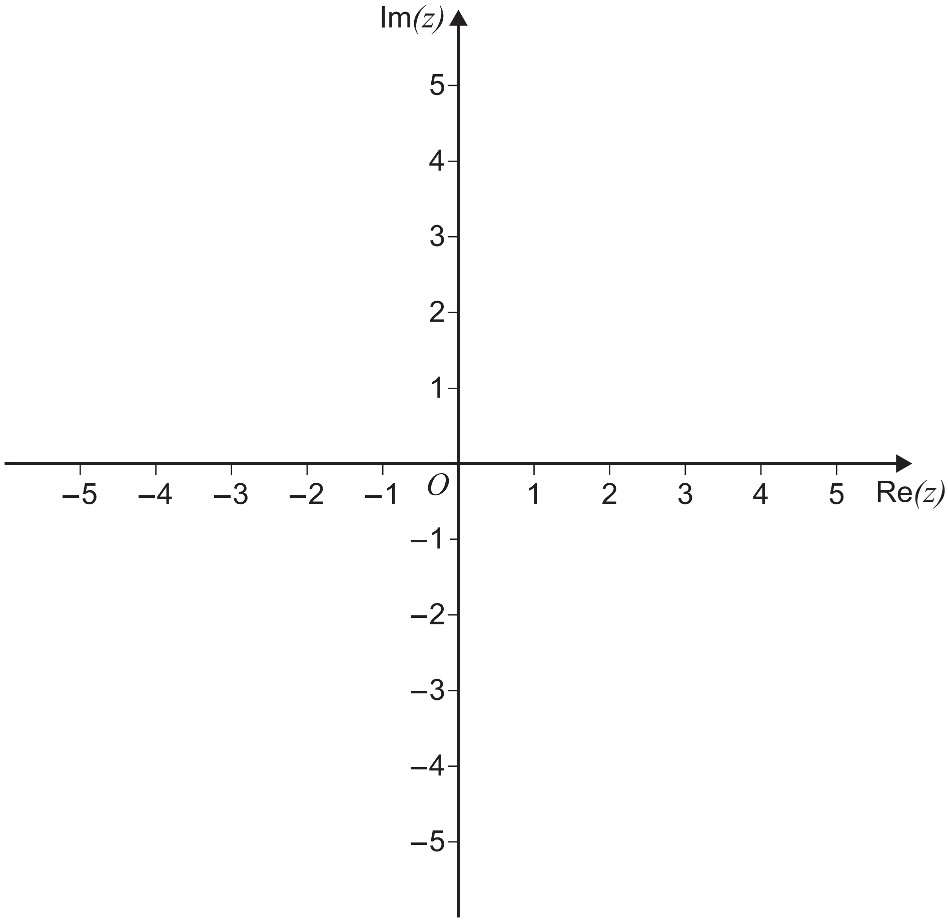
14 (a) Sketch, on the Argand diagram below, the locus of points satisfying the equation

$$|z - 3| = 2$$

[1 mark]



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14 (b) There is a unique complex number w that satisfies both

$$|w-3|=2 \quad \text{and} \quad \arg(w+1)=\alpha$$

where α is a constant such that $0<\alpha<\pi$

14 (b) (i) Find the value of α.

[2 marks]

14 (b) (ii) Express w in the form $r(\cos\theta + \mathrm{i}\sin\theta)$.

Give each of r and θ to two significant figures.

[4 marks]

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15 (a) Show that

$$\frac{1}{r+2} - \frac{1}{r+3} = \frac{1}{(r+2)(r+3)}$$

[1 mark]

15 (b) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} = \frac{n}{3(n+3)}$$

[3 marks]

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16 Two matrices **A** and **B** satisfy the equation

$$\mathbf{AB} = \boldsymbol{I} + 2\mathbf{A}$$

where $\boldsymbol{I}$ is the identity matrix and $\mathbf{B} = \begin{bmatrix} 3 & -2 \\ -4 & 8 \end{bmatrix}$

Find **A**.

[3 marks]

17 Find the exact solution to the equation

$$\sinh\theta(\sinh\theta + \cosh\theta) = 1$$

[4 marks]

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18 α, β and γ are the real roots of the cubic equation

$$x^3 + mx^2 + nx + 2 = 0$$

By considering $(\alpha - \beta)^2 + (\gamma - \alpha)^2 + (\beta - \gamma)^2$, prove that

$$m^2 \geq 3n$$

[4 marks]

19 A theme park has two zip wires.

Sarah models the two zip wires as straight lines using coordinates in metres.

The ends of one wire are located at $(0, 0, 0)$ and $(0, 100, -20)$

The ends of the other wire are located at $(10, 0, 20)$ and $(-10, 100, -5)$

19 (a) Use Sarah's model to find the shortest distance between the zip wires.

[7 marks]

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19 (b) State one way in which Sarah's model could be refined.

[1 mark]

END OF QUESTIONS

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