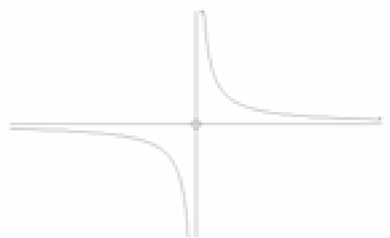
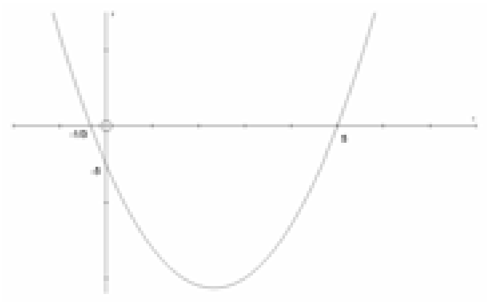
Mark Scheme 4721
June 2007

1	$(4x^2 + 20x + 25) – (x^2 - 6x + 9)$ $= 3x^2 + 26x + 16$ Alternative method using difference of two squares: $(2x + 5 + (x – 3))(2x + 5 – (x – 3))$ $= (3x + 2)(x + 8)$ $= 3x^2 + 26x + 16$	M1 A1 A1 3 **3**	Square one bracket to give an expression of the form $ax^2 + bx + c$ $(a \neq 0, b \neq 0, c \neq 0)$ One squared bracket fully correct All 3 terms of final answer correct M1 2 brackets with same terms but different signs A1 One bracket correctly simplified A1 All 3 terms of final answer correct
2 (a)(i) (ii) (b)	 Stretch Scale factor 8 in y direction **or** scale factor ½ in x direction	B1 B1 2 B1 1 B1 B1 2 **5**	Excellent curve for $\frac{1}{x}$ in either quadrant Excellent curve for $\frac{1}{x}$ in other quadrant **SR B1** Reasonably correct curves in 1st and 3rd quadrants Correct graph, minimum point at origin, symmetrical
3 (i) (ii)	$3\sqrt{20}$ or $3\sqrt{2}\ \sqrt{5} \times \sqrt{2}$ or $\sqrt{180}$ or $\sqrt{90} \times \sqrt{2}$ $= 6\sqrt{5}$ $10\sqrt{5} + 5\sqrt{5}$ $= 15\sqrt{5}$	M1 A1 2 M1 B1 A1 3 **5**	 Correctly simplified answer Attempt to change both surds to $\sqrt{5}$ One part correct and fully simplified cao

4 (i)	$(-4)^2 - 4 \times k \times k$ $= 16 - 4k^2$	M1 A1 2	Uses $b^2 - 4ac$ (involving k) $16 - 4k^2$
(ii)	$16 - 4k^2 = 0$ $k^2 = 4$ $k = 2$ or $k = -2$	M1 B1 B1 3 **5**	Attempts $b^2 - 4ac = 0$ (involving k) or attempts to complete square (involving k)
5 (i)	Length = 20 – 2x Area = x(20 – 2x) $= 20x - 2x^2$	M1 A1 2	Expression for length of enclosure in terms of x Correctly shows that area = $20x - 2x^2$ **AG**
(ii)	$\frac{dA}{dx} = 20 - 4x$ For max, 20 – 4x = 0 x = 5 only Area = 50	M1 M1 A1 A1 4 **6**	Differentiates area expression Uses $\frac{dy}{dx} = 0$
6	Let $y = (x + 2)^2$ $y^2 + 5y - 6 = 0$ (y + 6)(y - 1) = 0 y = -6 or y = 1 $(x + 2)^2 = 1$ x = -1 or x = -3	B1 M1 A1 M1 A1 A1 6 **6**	Substitute for $(x + 2)^2$ to get $y^2 + 5y - 6$ (= 0) Correct method to find roots Both values for y correct Attempt to work out x One correct value Second correct value and no extra real values
7 (a)	$f(x) = x + 3x^{-1}$ $f'(x) = 1 - 3x^{-2}$	M1 A1 A1 A1 4	Attempt to differentiate First term correct x^{-2} soi **www** Fully correct answer
(b)	$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$ When x = 4, $\frac{dy}{dx} = \frac{5}{2}\sqrt{4^3}$ $= 20$	M1 B1 B1 M1 A1 5 **9**	Use of differentiation to find gradient $\frac{5}{2}x^c$ $kx^{\frac{3}{2}}$ $\sqrt{4^3}$ soi **SR** If 0 scored for first 3 marks, award B1 if $\sqrt{4^n}$ correctly evaluated.

8 (i)	$(x + 4)^2 - 16 + 15$ $= (x + 4)^2 - 1$	B1 M1 A1 3	a = 4 15 – their a^2 cao in required form
(ii)	(-4, -1)	B1 ft B1 ft 2	Correct x coordinate Correct y coordinate
		M1 A1	Correct method to find roots -5, -3
(iii)	$x^2 + 8x + 15 > 0$ $(x + 5)(x + 3) > 0$ $x < -5, x > -3$	M1 A1 4 **9**	Correct method to solve quadratic inequality eg +ve quadratic graph x < -5, x > -3 (not wrapped, strict inequalities, no 'and')
9 (i)	$(x - 3)^2 - 9 + y^2 - k = 0$ $(x - 3)^2 + y^2 = 9 + k$ Centre (3, 0) $9 + k = 4^2$ $k = 7$	B1 B1 M1 A1 4	$(x-3)^2$ soi Correct centre Correct value for k (may be embedded) <u>Alternative method using expanded form:</u> Centre ($-g$, $-f$) M1 Centre (3, 0) A1 $4 = \sqrt{f^2 + g^2 - (-k)}$ M1 $k = 7$ A1
(ii)	$(3 - 3)^2 + y^2 = 16$ $y^2 = 16$ y = 4 Length of AB = $\sqrt{(-1-3)^2 + (0-4)^2}$ $= \sqrt{32}$ $= 4\sqrt{2}$	M1 A1 M1 A1 ft A1 5	Attempt to substitute x = 3 into original equation or their equation y = 4 (do not allow ± 4) Correct method to find line length using Pythagoras' theorem $\sqrt{32}$ or $\sqrt{16 + a^2}$ cao
(iii)	Gradient of AB = 1 or $\frac{a}{4}$ $y - 0 = m(x + 1)$ or $y - 4 = m(x - 3)$ $y = x + 1$	B1 ft M1 A1 3 **12**	 Attempts equation of straight line through their A or B with their gradient Correct equation in any form with simplified constants

10 (i)	(3x + 1)(x – 5) = 0 x = $\frac{-1}{3}$ or x = 5	M1 A1 A1 3	Correct method to find roots Correct brackets or formula Both values correct **SR B1** for x = 5 spotted **www**
(ii)		B1 B1 B1 ft 3	Positive quadratic (must be reasonably symmetrical) y intercept correct both x intercepts correct
(iii)	$\frac{dy}{dx}$ = 6x – 14 6x – 14 = 4 x = 3 On curve, when x = 3, y = -20 -20 = (4 x 3) + c c = -32	M1* M1* A1 A1 ft M1dep A1 6	Use of differentiation to find gradient of curve Equating their gradient expression to 4 Finding y co ordinate for their x value N.B. dependent on both previous M marks
	<u>Alternative method:</u> $3x^2$ – 14x – 5 = 4x + c $3x^2$ – 18x – 5 – c = 0 has one solution $b^2 - 4ac = 0$ $(-18)^2$ – (4 x 3 x (-5 –c)) = 0 c = -32	M1 B1 M1 M1 A1 A1 **12**	Equate curve and line (or substitute for x) Statement that only one solution for a tangent (may be implied by next line) Use of discriminant = 0 Attempt to use a, b, c from their equation Correct equation c = -32