RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4724/01**

MATHEMATICS

Core Mathematics 4

TUESDAY 23 JANUARY 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 It is given that

$$f(x) = \frac{x^2 + 2x - 24}{x^2 - 4x} \quad \text{for } x \neq 0,\ x \neq 4.$$

Express $f(x)$ in its simplest form. [3]

2 Find the exact value of $\int_1^2 x \ln x \, dx$. [5]

3 The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ relative to an origin O, where $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = -7\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$.

(i) Find the length of AB. [3]

(ii) Use a scalar product to find angle OAB. [3]

4 Use the substitution $u = 2x - 5$ to show that $\int_{\frac{5}{2}}^{3} (4x - 8)(2x - 5)^7 \, dx = \frac{17}{72}$. [5]

5 **(i)** Expand $(1 - 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3. [4]

(ii) Hence find the coefficient of x^3 in the expansion of $\left(1 - 3(x + x^3)\right)^{-\frac{1}{3}}$. [3]

6 **(i)** Express $\frac{2x + 1}{(x - 3)^2}$ in the form $\frac{A}{x - 3} + \frac{B}{(x - 3)^2}$, where A and B are constants. [3]

(ii) Hence find the exact value of $\int_4^{10} \frac{2x + 1}{(x - 3)^2} \, dx$, giving your answer in the form $a + b \ln c$, where a, b and c are integers. [4]

7 The equation of a curve is $2x^2 + xy + y^2 = 14$. Show that there are two stationary points on the curve and find their coordinates. [8]

8 The parametric equations of a curve are $x = 2t^2$, $y = 4t$. Two points on the curve are $P\ (2p^2, 4p)$ and $Q\ (2q^2, 4q)$.

(i) Show that the gradient of the normal to the curve at P is $-p$. [2]

(ii) Show that the gradient of the chord joining the points P and Q is $\frac{2}{p + q}$. [2]

(iii) The chord PQ is the normal to the curve at P. Show that $p^2 + pq + 2 = 0$. [2]

(iv) The normal at the point $R\ (8, 8)$ meets the curve again at S. The normal at S meets the curve again at T. Find the coordinates of T. [4]

9 **(i)** Find the general solution of the differential equation

$$\frac{\sec^2 y}{\cos^2(2x)}\frac{\mathrm{d}y}{\mathrm{d}x} = 2.$$ [7]

(ii) For the particular solution in which $y = \frac{1}{4}\pi$ when $x = 0$, find the value of y when $x = \frac{1}{6}\pi$. [3]

10 The position vectors of the points P and Q with respect to an origin O are $5\mathbf{i} + 2\mathbf{j} - 9\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ respectively.

(i) Find a vector equation for the line PQ. [2]

The position vector of the point T is $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(ii) Write down a vector equation for the line OT and show that OT is perpendicular to PQ. [4]

It is given that OT intersects PQ.

(iii) Find the position vector of the point of intersection of OT and PQ. [3]

(iv) Hence find the perpendicular distance from O to PQ, giving your answer in an exact form. [2]