

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4723

Core Mathematics 3

Thursday

8 JUNE 2006

Morning

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

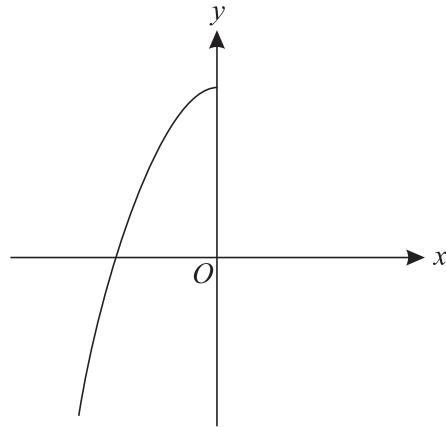
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

- 1 Find the equation of the tangent to the curve $y = \sqrt{4x + 1}$ at the point (2, 3). [5]
- 2 Solve the inequality $|2x - 3| < |x + 1|$. [5]
- 3 The equation $2x^3 + 4x - 35 = 0$ has one real root.
- (i) Show by calculation that this real root lies between 2 and 3. [3]
- (ii) Use the iterative formula
- $$x_{n+1} = \sqrt[3]{17.5 - 2x_n},$$
- with a suitable starting value, to find the real root of the equation $2x^3 + 4x - 35 = 0$ correct to 2 decimal places. You should show the result of each iteration. [3]
- 4 It is given that $y = 5^{x-1}$.
- (i) Show that $x = 1 + \frac{\ln y}{\ln 5}$. [2]
- (ii) Find an expression for $\frac{dx}{dy}$ in terms of y . [2]
- (iii) Hence find the exact value of the gradient of the curve $y = 5^{x-1}$ at the point (3, 25). [2]
- 5 (i) Write down the identity expressing $\sin 2\theta$ in terms of $\sin \theta$ and $\cos \theta$. [1]
- (ii) Given that $\sin \alpha = \frac{1}{4}$ and α is acute, show that $\sin 2\alpha = \frac{1}{8}\sqrt{15}$. [3]
- (iii) Solve, for $0^\circ < \beta < 90^\circ$, the equation $5 \sin 2\beta \sec \beta = 3$. [3]

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The diagram shows the graph of $y = f(x)$, where

$$f(x) = 2 - x^2, \quad x \leq 0.$$

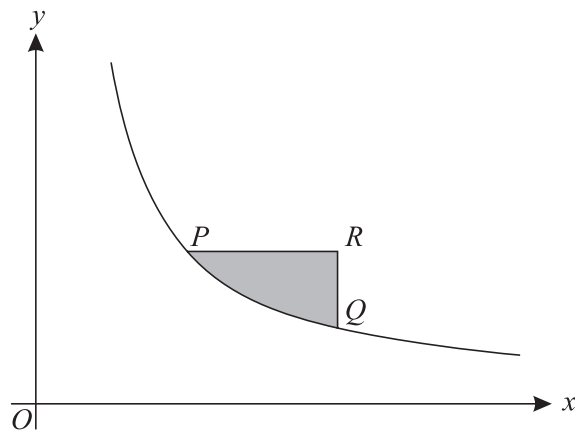
(i) Evaluate $ff(-3)$. [3]

(ii) Find an expression for $f^{-1}(x)$. [3]

(iii) Sketch the graph of $y = f^{-1}(x)$. Indicate the coordinates of the points where the graph meets the axes. [3]

7 (a) Find the exact value of $\int_1^2 \frac{2}{(4x-1)^2} dx$. [4]

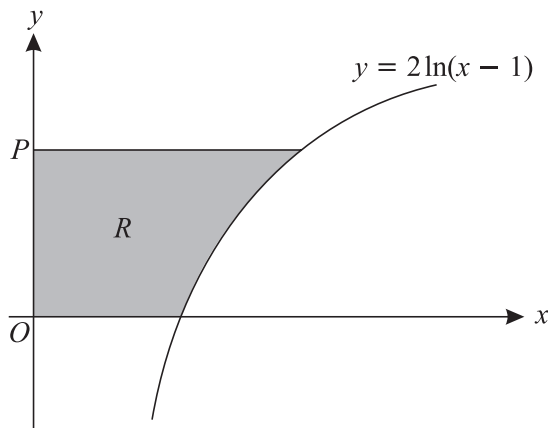
(b)



The diagram shows part of the curve $y = \frac{1}{x}$. The point P has coordinates $(a, \frac{1}{a})$ and the point Q has coordinates $(2a, \frac{1}{2a})$, where a is a positive constant. The point R is such that PR is parallel to the x -axis and QR is parallel to the y -axis. The region shaded in the diagram is bounded by the curve and by the lines PR and QR . Show that the area of this shaded region is $\ln(\frac{1}{2}e)$. [6]

- 8 (i) Express $5 \cos x + 12 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]
- (ii) Hence give details of a pair of transformations which transforms the curve $y = \cos x$ to the curve $y = 5 \cos x + 12 \sin x$. [3]
- (iii) Solve, for $0^\circ < x < 360^\circ$, the equation $5 \cos x + 12 \sin x = 2$, giving your answers correct to the nearest 0.1° . [5]

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The diagram shows the curve with equation $y = 2 \ln(x - 1)$. The point P has coordinates $(0, p)$. The region R , shaded in the diagram, is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The units on the axes are centimetres. The region R is rotated completely about the **y-axis** to form a solid.

- (i) Show that the volume, $V \text{ cm}^3$, of the solid is given by

$$V = \pi(e^p + 4e^{\frac{1}{2}p} + p - 5). \quad [8]$$

- (ii) It is given that the point P is moving in the positive direction along the y -axis at a constant rate of 0.2 cm min^{-1} . Find the rate at which the volume of the solid is increasing at the instant when $p = 4$, giving your answer correct to 2 significant figures. [5]