Mark Scheme 4723
January 2006

1

Obtain integral of form $k \ln x$	**M1**		[any non-zero constant k; or equiv such as $k \ln 3x$]
Obtain $3 \ln 8 - 3 \ln 2$	**A1**		[or exact equiv]
Attempt use of at least one relevant log property	**M1**		[would be earned by initial $\ln x^3$]
Obtain $3 \ln 4$ or $\ln 8^3 - \ln 2^3$ and hence $\ln 64$	**A1**	**4**	[**AG**; with no errors]

2

Attempt use of identity linking $\sec^2\theta$, $\tan^2\theta$ and 1	**M1**		[to write eqn in terms of $\tan\theta$]
Obtain $\tan^2\theta - 4\tan\theta + 3 = 0$	**A1**		[or correct unsimplified equiv]
Attempt solution of quadratic eqn to find two values of $\tan\theta$	**M1**		[any 3 term quadratic eqn in $\tan\theta$]
Obtain at least two correct answers	**A1**		[after correct solution of eqn]
Obtain all four of 45, 225, 71.6, 251.6	**A1**	**5**	[allow greater accuracy or angles to nearest degree – and no other answers between 0 and 360]

---

3 (a)

Attempt use of product rule	**M1**		[involving … + …]
Obtain $2x(x+1)^6$ …	**A1**		
Obtain … + $6x^2(x+1)^5$	**A1**	**3**	[or equivs; ignore subsequent attempt at simplification]

(b)

Attempt use of quotient rule	**M1**		[or, with adjustment, product rule; allow u / v confusion]
Obtain $\dfrac{(x^2-3)2x-(x^2+3)2x}{(x^2-3)^2}$	**A1**		[or equiv]
Obtain –3	**A1**	**3**	[from correct derivative only]

---

4 (i)

State $y \le 2$	**B1**	**1**	[or equiv; allow <; allow any letter or none]

(ii)

Show correct process for composition of functions	**M1**		[numerical or algebraic]
Obtain 0 and hence 2	**A1**	**2**	[and no other value]

(iii)

State a range of values with 2 as one end-point	**M1**		[continuous set, not just integers]
State $0 < k \le 2$	**A1**	**2**	[with correct < and ≤ now]

5

Obtain integral of form $k(1-2x)^6$	**M1**		[any non-zero constant k]
Obtain correct $-\frac{1}{12}(1-2x)^6$	**A1**		[or unsimplified equiv; allow + c]
Use limits to obtain $\frac{1}{12}$	**A1**		[or exact (unsimplified) equiv]
Obtain integral of form $k\,e^{2x-1}$	**M1**		[or equiv; any non-zero constant k]
Obtain correct $\frac{1}{2}e^{2x-1} - x$	**A1**		[or equiv; allow + c]
Use limits to obtain $-\frac{1}{2}e^{-1}$	**A1**		[or exact (unsimplified) equiv]
Show correct process for finding required area	**M1**		[at any stage of solution; if process involves two definite integrals, second must be negative]
Obtain $\frac{1}{12} + \frac{1}{2}e^{-1}$	**A1**	**8**	[or exact equiv; no + c]

6 (a) Either: State proportion $\frac{440}{275}$ **B1**

Attempt calculation involving proportion **M1** [involving multn and X value]

Obtain 704 **A1 3**

Or: Use formula of form $275\,e^{kt}$ or $275\,a^{t}$ **M1** [or equiv]

Obtain $k = 0.047$ or $a = \sqrt[10]{1.6}$ **A1** [or equiv]

Obtain 704 **A1 (3)** [allow ±0.5]

(b)(i) Attempt correct process involving logarithm **M1** [or equiv including systematic trial and improvement attempt]

Obtain $\ln\frac{20}{80} = -0.02t$ **A1** [or equiv]

Obtain 69 **A1 3** [or greater accuracy; scheme for T&I: M1A2]

(ii) Differentiate to obtain $k\,e^{-0.02t}$ **M1** [any constant k different from 80]

Obtain $-1.6\,e^{-0.02t}$ (or $1.6\,e^{-0.02t}$) **A1** [or unsimplified equiv]

Obtain 0.88 **A1 3** [or greater accuracy; allow –0.88]

---

7 (i) Sketch curve showing (at least) translation in x direction **M1** [either positive or negative]

Show correct sketch with one of 2 and 3π indicated **A1**

… and with other one of 2 and 3π indicated **A1 3**

(ii) Draw straight line through O with positive gradient **B1 1** [label and explanation not required]

(iii) Attempt calculations using 1.8 and 1.9 **M1** [allow here if degrees used]

Obtain correct values and indicate change of sign **A1 2** [or equiv; x = 1.8: LHS = 1.93, diff = 0.13; x = 1.9: LHS = 1.35, diff = -0.55; radians needed now]

(iv) Obtain correct first iterate 1.79 or 1.78 **B1** [or greater accuracy]

Attempt correct process to produce at least 3 iterates **M1**

Obtain 1.82 **A1** [answer required to exactly 2 d.p.; $2 \to 1.7859 \to 1.8280 \to 1.8200$; SR: answer 1.82 only - B2]

Attempt rearrangement of $3\cos^{-1}(x-1) = x$ or of $x = 1 + \cos(\frac{1}{3}x)$ **M1** [involving at least two steps]

Obtain required formula or equation respectively **A1 5**

8	**(i)**	Differentiate to obtain $kx(5-x^2)^{-1}$	**M1**	[any non-zero constant]
		Obtain correct $-2x(5-x^2)^{-1}$	**A1**	[or equiv]
		Obtain -4 for value of derivative	**A1**	
		Attempt equation of straight line through (2, 0) with numerical value of gradient obtained from attempt at derivative	**M1**	[not for attempt at eqn of normal]
		Obtain $y = -4x + 8$	**A1** **5**	[or equiv]
	(ii)	State or imply $h = \frac{1}{2}$	**B1**	
		Attempt calculation involving attempts at y values	**M1**	[addition with each of coefficients 1, 2, 4 occurring at least once]
		Obtain $k(\ln 5 + 4\ln 4.75 + 2\ln 4 + 4\ln 2.75 + \ln 1)$	**A1**	[or equiv perhaps with decimals; any constant k]
		Obtain 2.44	**A1** **4**	[allow ± 0.01]
	(iii)	Attempt difference of two areas	**M1**	[allow if area of their triangle < area A]
		Obtain $8 - 2.44$ and hence 5.56	**A1√** **2**	[following their tangent and area of A providing answer positive]

9	**(i)**	State $\sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	**B1**	
		Use at least one of $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$	**B1**	
		Attempt complete process to express in terms of $\sin\theta$	**M1**	[using correct identities]
		Obtain $3\sin\theta - 4\sin^3\theta$	**A1** **4**	[**AG**; all correctly obtained]
	(ii)	State 3	**B1**	
		Obtain expression involving $\sin 10\alpha$	**M1**	[allow θ/α confusion]
		Obtain 9	**A1** **3**	[and no other value]
	(iii)	Recognise cosec 2β as $\frac{1}{\sin 2\beta}$	**B1**	[allow θ/β confusion]
		Attempt to express equation in terms of $\sin 2\beta$ only	**M1**	[or equiv involving $\cos 2\beta$]
		Attempt to find non-zero value of $\sin 2\beta$	**M1**	[or of $\cos 2\beta$]
		Obtain at least $\sin 2\beta = \sqrt{\frac{5}{12}}$	**A1**	[or equiv, exact or approx]
		Attempt correct process to find two values of β	**M1**	[provided equation is $\sin 2\beta = k$; or equiv with $\cos 2\beta$]
		Obtain 20.1, 69.9	**A1** **6**	[and no others between 0 and 90]