

Mark Scheme 4723  
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1	(i)	State $f(x) \leq 10$	B1	1 [Any equiv but must be or imply $\leq$ ]
	(ii)	Attempt correct process for composition of functions Obtain 6 or correct expression for $ff(x)$ Obtain $-71$	M1 A1 A1	[whether algebraic or numerical]  <b>3</b>
2		<u>Either</u> Obtain $x = 0$ Form linear equation with signs of $6x$ and $x$ different State $6x - 1 = -x + 1$  Obtain $\frac{2}{7}$ and no other non-zero value	B1 M1 A1 A1	[ignoring errors in working] [ignoring other sign errors] [or correct equiv with or without brackets] <b>4</b> [or exact equiv]
	<u>Or</u>	Obtain $36x^2 - 12x + 1 = x^2 - 2x + 1$ Attempt to solve quadratic equation  Obtain $\frac{2}{7}$ and no other non-zero value Obtain 0	B1 M1 A1 B1	[or equiv] [as far as factorisation or subn into formula] [or exact equiv] <b>(4)</b> [ignoring errors in working]
3	(i)	Attempt solution involving (natural) logarithm  Obtain $-0.017t = \ln \frac{25}{180}$  Obtain 116	M1 A1 A1	[or equiv]  <b>3</b> [or greater accuracy rounding to 116]
	(ii)	Differentiate to obtain $ke^{-0.017t}$  Obtain correct $-3.06e^{-0.017t}$  Obtain 1.2	M1 A1 A1	[any constant $k$ different from 180; solution must involve differentiation] [or unsimplified equiv; accept + or -] <b>3</b> [or greater accuracy; accept + or - answer]
4	(a)	State or imply $\int \pi y^2 dx$  Integrate to obtain $k \ln x$  Obtain $4\pi \ln x$ or $4 \ln x$  Obtain $4\pi \ln 5$	B1 M1 A1 A1	[any constant $k$ , involving $\pi$ or not; or equiv such as $k \ln 4x$ ] [or equiv] <b>4</b> [or similarly simplified equiv]

	<p><b>(b)</b> Attempt calculation involving attempts at <math>y</math> values</p> <p>Attempt <math>\frac{1}{3} \times 1(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)</math></p> <p>Obtain <math>\frac{1}{3}(\sqrt{2} + 4\sqrt{5} + 2\sqrt{10} + 4\sqrt{17} + \sqrt{26})</math></p> <p>Obtain 12.758</p>	<p><b>M1</b> [with each of 1, 4, 2 present at least once as coefficients]</p> <p><b>M1</b> [with attempts at five <math>y</math> values]</p> <p><b>A1</b> [or exact equiv or decimal equivalents]</p> <p><b>A1</b> <b>4</b> [or greater accuracy]</p>
5	<p><b>(i)</b> Obtain <math>R = \sqrt{13}</math>, or 3.6 or 3.61 or greater accuracy</p> <p>Attempt recognisable process for finding <math>\alpha</math></p> <p>Obtain <math>\alpha = 33.7</math></p>	<p><b>B1</b></p> <p><b>M1</b> [allow sine/cosine muddles]</p> <p><b>A1</b> <b>3</b> [or greater accuracy]</p>
	<p><b>(ii)</b> Attempt to find at least one value of <math>\theta + \alpha</math></p> <p>Obtain value rounding to 76 or 104</p> <p>Subtract their <math>\alpha</math> from at least one value</p> <p>Obtain one value rounding to 42 or 43, or to 70</p> <p>Obtain other value 42.4 or 70.2</p>	<p><b>*M1</b></p> <p><b>A1</b>✓ [following their <math>R</math>]</p> <p><b>M1</b> [dependent on <b>*M</b>]</p> <p><b>A1</b></p> <p><b>A1</b> <b>5</b> [or greater accuracy; no other answers between 0 and 360; ignore answers outside 0 to 360]</p>
6	<p><b>(a)</b> Attempt use of product rule</p> <p>Obtain <math>\ln x + 1</math></p> <p>Equate attempt at first derivative to zero and obtain value involving <math>e</math></p> <p>Obtain <math>e^{-1}</math></p>	<p><b>*M1</b></p> <p><b>A1</b> [or unsimplified equiv]</p> <p><b>M1</b> [dependent on <b>*M</b>]</p> <p><b>A1</b> <b>4</b> [or exact equiv]</p>
	<p><b>(b)</b> Attempt use of quotient rule</p> <p>Obtain <math>\frac{(4x-c)4 - 4(4x+c)}{(4x-c)^2}</math></p> <p>Show that first derivative cannot be zero</p>	<p><b>M1</b> [or equiv using product rule or ...]</p> <p><b>A1</b> [or equiv]</p> <p><b>A1</b> <b>3</b> [<b>AG</b>; derivative must be correct]</p>
7	<p><b>(i)</b> State <math>2\cos^2 x - 1</math></p>	<p><b>B1</b> <b>1</b></p>
	<p><b>(ii)</b> Attempt to express left hand side in terms of <math>\cos x</math></p> <p>Identify <math>\frac{1}{\cos x}</math> as <math>\sec x</math></p>	<p><b>M1</b> [using expression of form <math>a\cos^2 x + b</math>]</p> <p><b>M1</b> [maybe implied]</p>

		Confirm result	<b>A1</b>	<b>3</b> [AG; necessary detail required]
	<b>(iii)</b>	Use identity $\sec^2 x = 1 + \tan^2 x$ Attempt solution of quadratic equation in $\tan x$ Obtain $2 \tan^2 x + 3 \tan x - 9 = 0$ and hence $\tan x = -3, \frac{3}{2}$ Obtain at least two of 0.983, 4.12, 1.89, 5.03 (or of $0.313\pi, 1.31\pi, 0.602\pi, 1.60\pi$ ) Obtain all four solutions	<b>B1</b> <b>M1</b> <b>A1</b> <b>A1</b> <b>A1</b>	[or equiv]  [allow answers with only 2 s.f.; allow greater accuracy; allow $0.983 + \pi, 1.89 + \pi$ allow degrees: 56, 236, 108, 288] <b>5</b> [now with at least 3 s.f.; must be radians; no other solutions in the range $0 - 2\pi$ , ignore solutions outside range $0 - 2\pi$ ]
<b>8</b>	<b>(i)</b>	Attempt relevant calculations with 5.2 and 5.3 Obtain correct values  Conclude appropriately	<b>M1</b> <b>A1</b> <b>A1</b>	$x$ $y_1$ $y_2$ $y_1 - y_2$ 5.2            2.83            2.87            -0.04 5.3            2.89            2.88            0.006 <b>3</b> [AG; comparing y values or noting sign change in difference in y values or equiv]
	<b>(ii)</b>	Equate expressions and attempt rearrangement to $x =$ Obtain $x = \frac{5}{3} \ln(3x + 8)$	<b>M1</b> <b>A1</b>	<b>2</b> [AG; necessary detail required]
	<b>(iii)</b>	Obtain correct first iterate Carry out correct process to find at least two iterates in all Obtain 5.29	<b>B1</b> <b>M1</b> <b>A1</b>	<b>3</b> [must be exactly 2 decimal places;  5.2→5.2687→5.2832→5.2863→5.2869; 5.25→5.2793→5.2855→5.2868→5.2870; 5.3→5.2898→5.2877→5.2872→5.2871]
	<b>(iv)</b>	Obtain integral of form $k(3x + 8)^{\frac{4}{3}}$ Obtain integral of form $k e^{\frac{1}{5}x}$	<b>M1</b> <b>M1</b>	

		Obtain $\frac{1}{4}(3x+8)^{\frac{4}{3}} - 5e^{\frac{1}{5}x}$	<b>A1</b>	[or equiv]
		Apply limits 0 and their answer to <b>(iii)</b>	<b>M1</b>	[applied to difference of two integrals]
		Obtain 3.78	<b>A1</b>	<b>5</b> [or greater accuracy]
<b>9</b>	<b>(i)</b>	Indicate stretch and (at least one) translation	<b>M1</b>	[... in general terms]
		State translation by 7 units in negative $x$ direction	<b>A1</b>	[or equiv; using correct terminology]
		State stretch in $x$ direction with factor $1/m$	<b>A1</b>	[must follow the translation by 7; or equiv; using correct terminology]
	<b>(i)</b>	Indicate translation by 4 units in negative $y$ direction	<b>B1</b>	<b>4</b> [or equiv; at any stage; the two translations may be combined]
	<b>(ii)</b>	Refer to each $y$ value being image of unique $x$ value	<b>B1</b>	[or equiv]
		Attempt correct process for finding inverse	<b>M1</b>	
		Obtain expression involving $(x+4)^2$ or $(y+4)^2$	<b>M1</b>	
		Obtain $\frac{(x+4)^2 - 7}{m}$	<b>A1</b>	<b>4</b> [or equiv]
	<b>(iii)</b>	Refer to fact that curves are reflections of each other in line $y = x$	<b>B1</b>	[or equiv]
		Attempt arrangement of either $f(x) = x$ or $f^{-1}(x) = x$	<b>M1</b>	
		Apply discriminant to resulting quadratic equation	<b>M1</b>	
		Obtain $(m-2)(m-14) < 0$	<b>A1</b>	[or equiv]
		Obtain $2 < m < 14$	<b>A1</b>	<b>5</b>